Knowledge Base Partitioning

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FORMAL REASONING GROUP

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Thanks to:

- Sheila A. McIlraith (Stanford/KSL)
- Vinay K. Chaudhri (SRI)

Motivating Problem

- **Exploiting structure inherent in a set of logical axioms for efficient reasoning.**

Contents

A partitioning and its intersection graph

$\mathcal{A}$

- $\neg ok\_pump \vee \neg on\_pump \vee water$
- $\neg man\_fill \vee water$
- $\neg man\_fill \vee \neg on\_pump$
- $man\_fill \vee on\_pump$
- $\neg water \vee \neg ok\_boiler \vee \neg on\_boiler \vee steam$
- $water \vee \neg steam$
- $ok\_boiler \vee \neg steam$
- $on\_boiler \vee \neg steam$
- $\neg steam \vee \neg coffee \vee hot\_drink$
- $coffee \vee teabag$
- $\neg steam \vee \neg teabag \vee hot\_drink$

$\mathcal{A}_1$

(1) $\neg ok\_pump \vee \neg on\_pump \vee water$

(2) $\neg man\_fill \vee water$

(3) $\neg man\_fill \vee \neg on\_pump$

(4) $man\_fill \vee on\_pump$

— $water$ —

$\mathcal{A}_2$

(5) $\neg water \vee \neg ok\_boiler \vee \neg on\_boiler \vee steam$

(6) $water \vee \neg steam$

(7) $ok\_boiler \vee \neg steam$

(8) $on\_boiler \vee \neg steam$

— $steam$ —

$\mathcal{A}_3$

(9) $\neg steam \vee \neg coffee \vee hot\_drink$

(10) $coffee \vee teabag$

(11) $\neg steam \vee \neg teabag \vee hot\_drink$

A partitioning of $\mathcal{A}$ and its intersection graph.

A Message-Passing Proof

Using MP to prove *hot_drink* **after asserting** *ok_pump* **(12) in** $\mathcal{A}_1$ **and** *ok_boiler* **(13),** *on_boiler* **(14) in** $\mathcal{A}_2$**.**

$\mathcal{A}_1$

- (1) $\neg ok\_pump \vee \neg on\_pump \vee water$
- (2) $\neg man\_fill \vee water$
- (3) $\neg man\_fill \vee \neg on\_pump$
- (4) $man\_fill \vee on\_pump$

water ($\mathcal{A}_1$ – $\mathcal{A}_2$)

$\mathcal{A}_2$

- (5) $\neg water \vee \neg ok\_boiler \vee \neg on\_boiler \vee steam$
- (6) $water \vee \neg steam$
- (7) $ok\_boiler \vee \neg steam$
- (8) $on\_boiler \vee \neg steam$

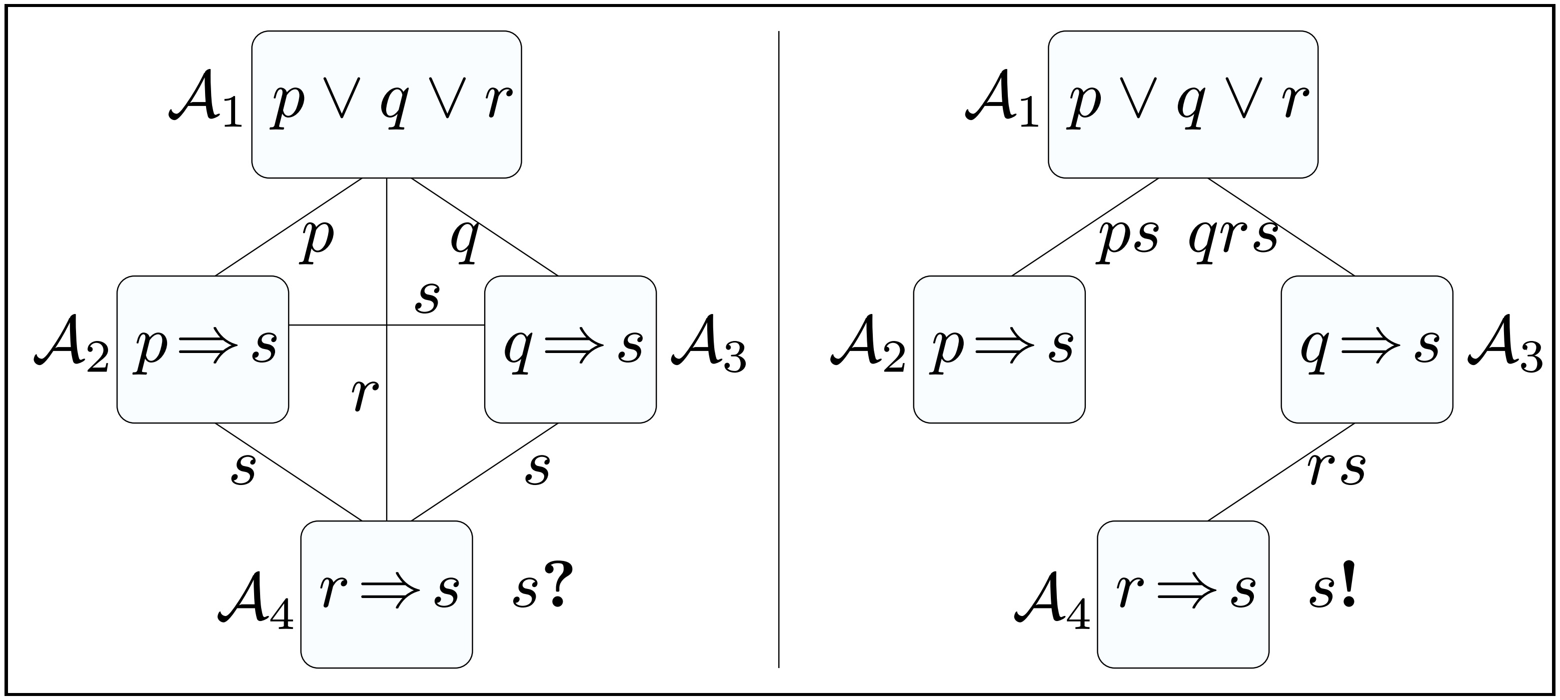
steam ($\mathcal{A}_2$ – $\mathcal{A}_3$)

$\mathcal{A}_3$

- (9) $\neg steam \vee \neg coffee \vee hot\_drink$
- (10) $coffee \vee teabag$
- (11) $\neg steam \vee \neg teabag \vee hot\_drink$

Resolve	Generating	
(2),(4)	$on\_pump \vee water$	**(m1)**
(m1),(1)	$ok\_pump \vee water$	**(m2)**
(m2),(12)	$water$	**(m3)**
clause *water* **passed from** $\mathcal{A}_1$ **to** $\mathcal{A}_2$		
(m3),(5)	$ok\_boiler \wedge on\_boiler \Rightarrow steam$	**(m4)**
(m4),(13)	$\neg on\_boiler \vee steam$	**(m5)**
(m5),(14)	$steam$	**(m6)**
clause *steam* **passed from** $\mathcal{A}_2$ **to** $\mathcal{A}_3$		
(9),(10)	$\neg steam \vee teabag \vee hot\_drink$	**(m7)**
(m7),(11)	$\neg steam \vee hot\_drink$	**(m8)**
(m8),(m6)	hot_drink	**(m9)**

Graphs with Cycles



An intersection graph before (left) and after (right) applying BREAK-CYCLES.

Soundness and Completeness of MP

Craig's Interpolation Theorem: ***If*** $\alpha \vdash \beta$***, then there is a formula*** γ ***involving only symbols common to both*** α ***and*** β***, such that*** $\alpha \vdash \gamma$ ***and*** $\gamma \vdash \beta$***.***

Theorem 1 ***Message-Passing with Break-Cycles is sound and complete, if the reasoning procedure in each partition is complete for consequence finding.***

(e.g., $\alpha = \mathcal{A}_1$, $\beta = \mathcal{A}_2 \Rightarrow Q$**)**

A Good partitioning

(Amir & McIlraith 2000): Minimize

1. **The total number of symbols contained in all links to/from node i.**
2. **The number of axioms in each partition.**
3. **p - the number of partitions.**

Contents

1. **Theorem proving with partitions**
2. **Automatic partitioning:**
   - **Algorithm SPLIT**
   - **Algorithm TRIANGULATE**
3. **Preliminary results:**
   - **(some of) SRI's HPKB knowledge base**
   - **(some of) Cyc's Spatial Axioms**

Optimal partitioning

1. **Finding optimal partitioning is NP-hard (Arnborg et al. 1987).**
2. **Can the optimal be approximated with constant factor in polynomial time? – Open question**
3. **Many partitioning algorithms so far: (Lagergren and Arnborg 1991), (Reed 1992), (Kloks 1994), (Bodlaender & Kloks 1991), (Bodlaender 1996), (Becker & Geiger 1996), (Shoikhet & Geiger 1997).**

Automated Partitioning using SPLIT

$\mathcal{A}$

$\neg ok\_pump \vee \neg on\_pump \vee water$

$\neg man\_fill \vee \neg on\_pump$

$\neg water \vee \neg ok\_boiler \vee \neg on\_boiler \vee steam$

$ok\_boiler \vee \neg steam$

$\neg steam \vee \neg coffee \vee hot\_drink$

$\neg steam \vee \neg teabag \vee hot\_drink$

$\neg man\_fill \vee water$

$man\_fill \vee on\_pump$

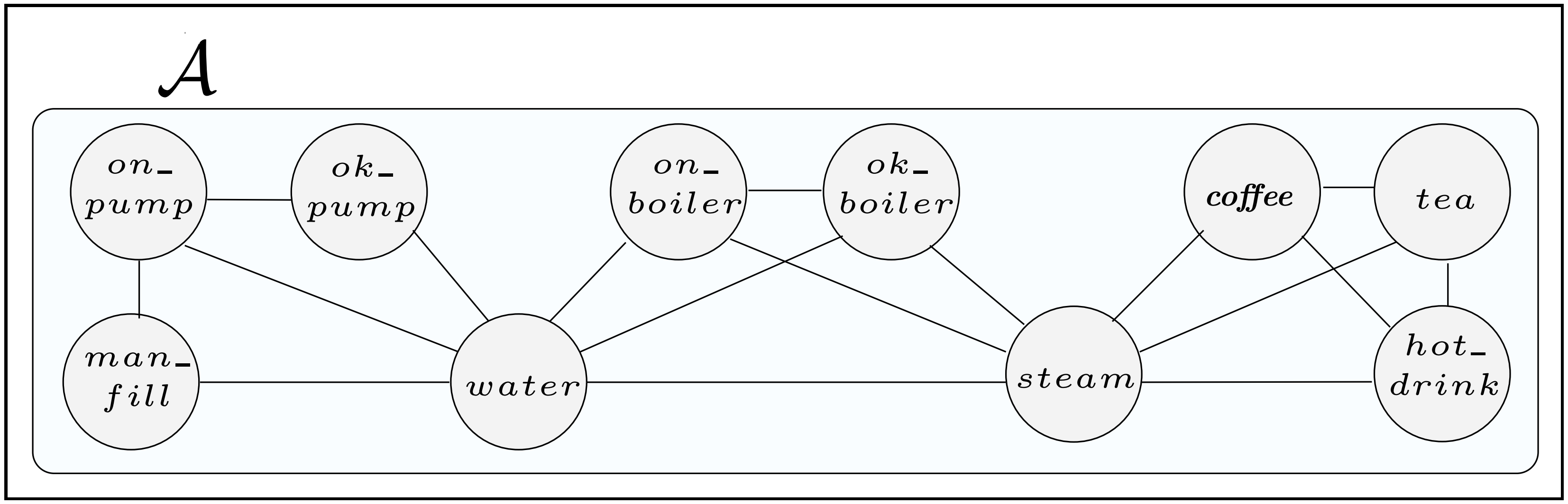
$water \vee \neg steam$

$on\_boiler \vee \neg steam$

$coffee \vee teabag$

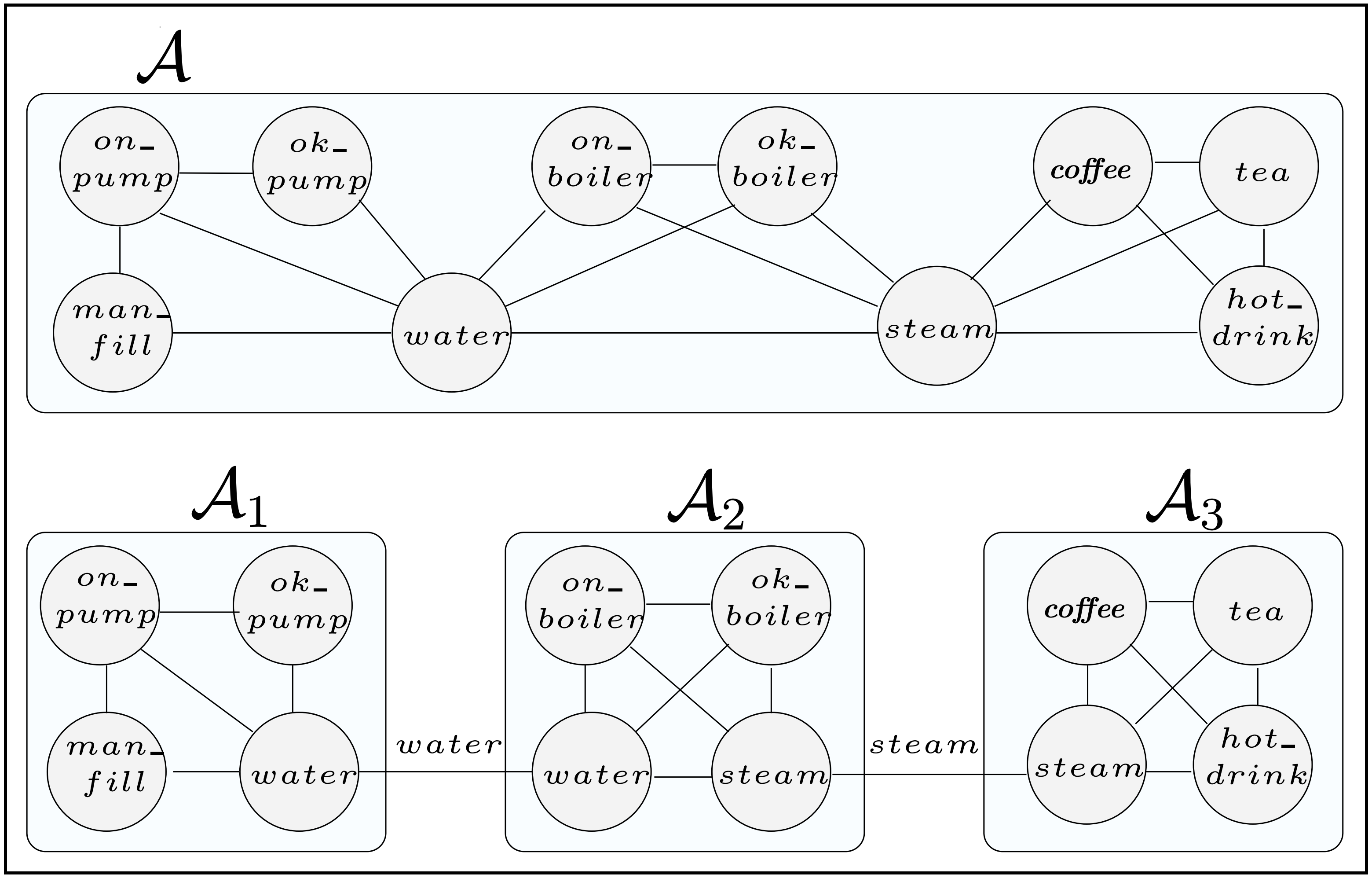
$\mathcal{A}$'s theory

Automated Partitioning using SPLIT



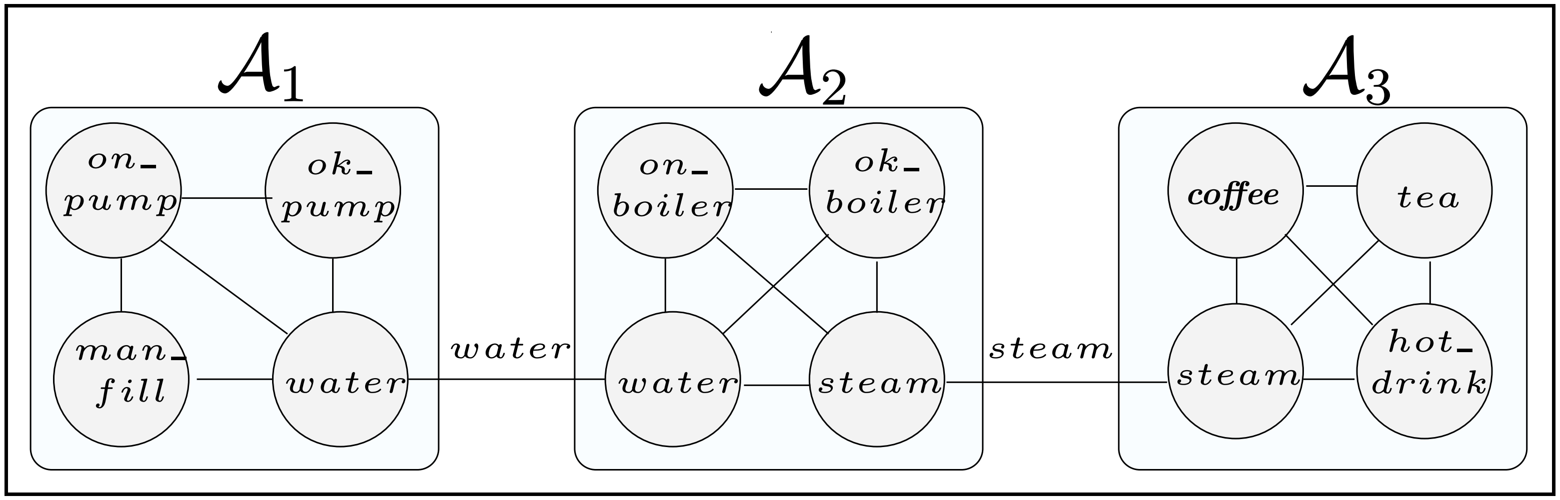
$\mathcal{A}$'s symbols graph

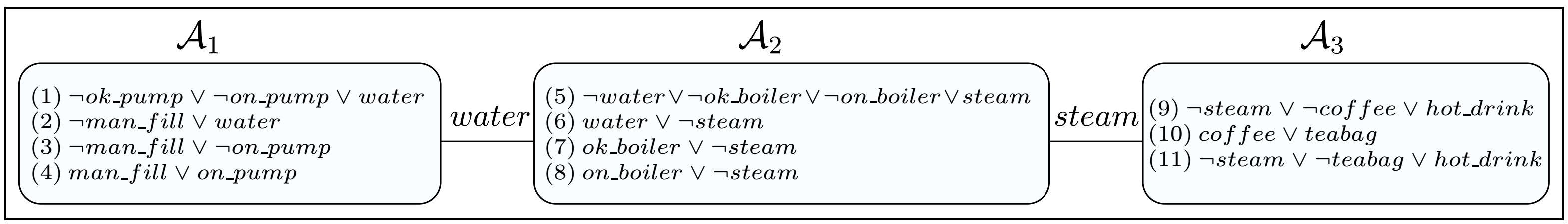
Automated Partitioning using SPLIT



Decomposing $\mathcal{A}$'s symbols graph with *vertex min-cut*

Automated Partitioning using SPLIT

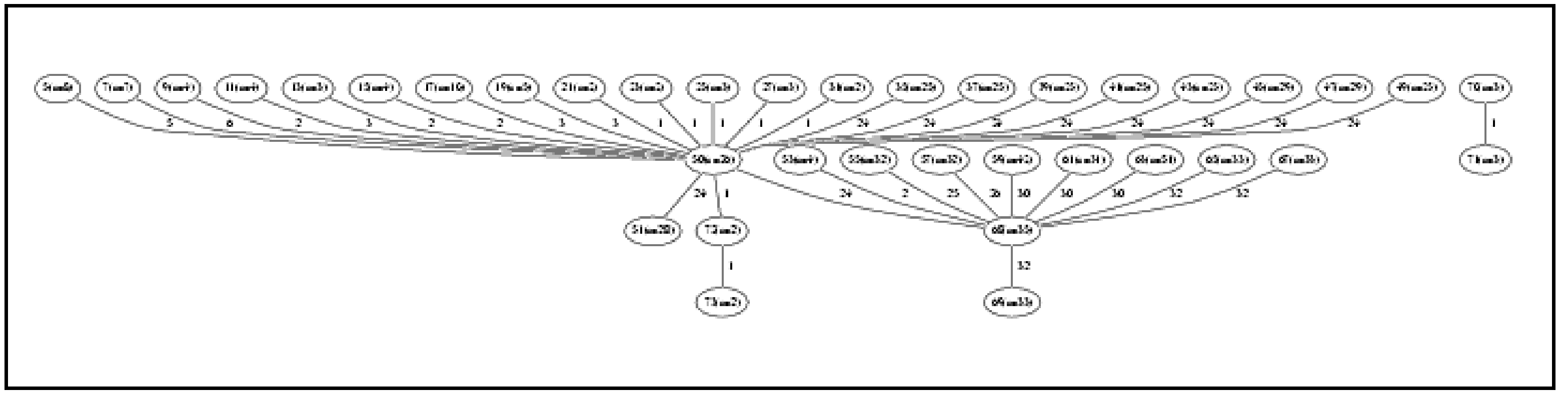




Analysis of SPLIT

- **At each iteration, SPLIT computes a minimum vertex separator.**
- **SPLIT is a greedy, recursive algorithm.**
- **It takes time $O(|V|^{\frac{5}{2}} * |E|)$: linear dependency on the number of clauses, and small polynomial dependency on the number of propositional symbols.**
- **The algorithm should stop only when the subgraph is a clique (assuming $f_{SAT}(m) = O(2^{\alpha m})$).**

Using SPLIT on (some of) Cyc's Spatial Axioms



223 **axioms over** 142 **symbols.**

Time for finding the decomposition: 5 seconds.

Largest partition: 51 symbols

Largest link: 32 symbols

Contents

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   - **(some of) Cyc's Spatial Axioms**

Automated Partitioning using TRIANG.

$\mathcal{A}$

$\neg ok\_pump \vee \neg on\_pump \vee water$

$\neg man\_fill \vee \neg on\_pump$

$\neg water \vee \neg ok\_boiler \vee \neg on\_boiler \vee steam$

$ok\_boiler \vee \neg steam$

$\neg steam \vee \neg coffee \vee hot\_drink$

$\neg steam \vee \neg teabag \vee hot\_drink$

$\neg man\_fill \vee water$

$man\_fill \vee on\_pump$

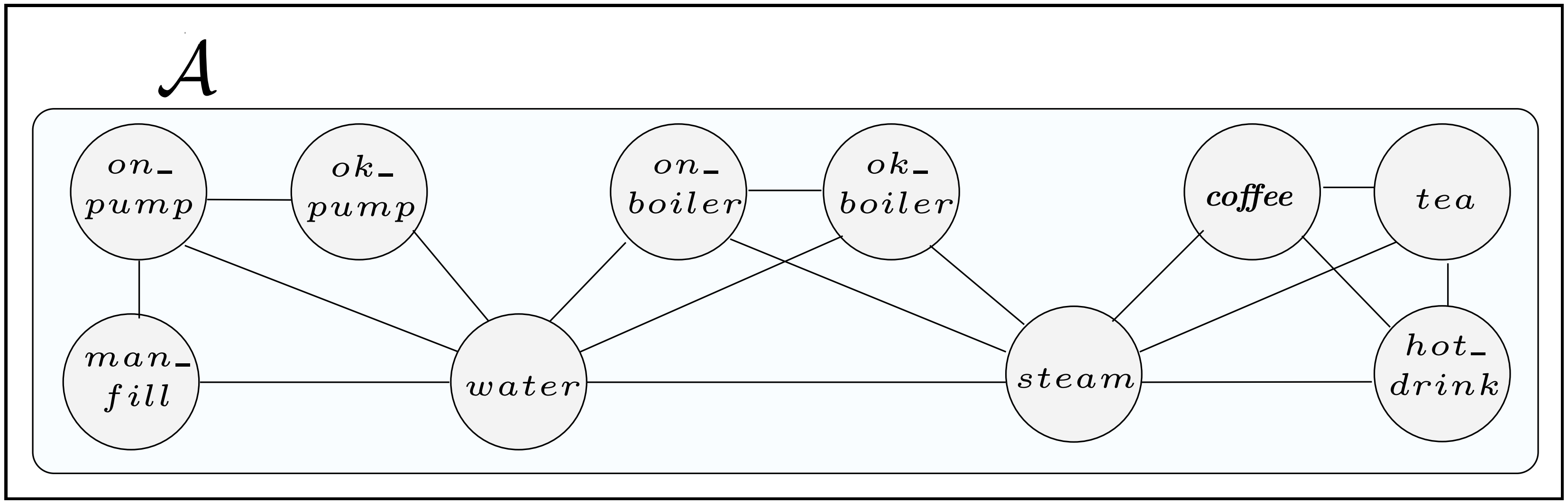
$water \vee \neg steam$

$on\_boiler \vee \neg steam$

$coffee \vee teabag$

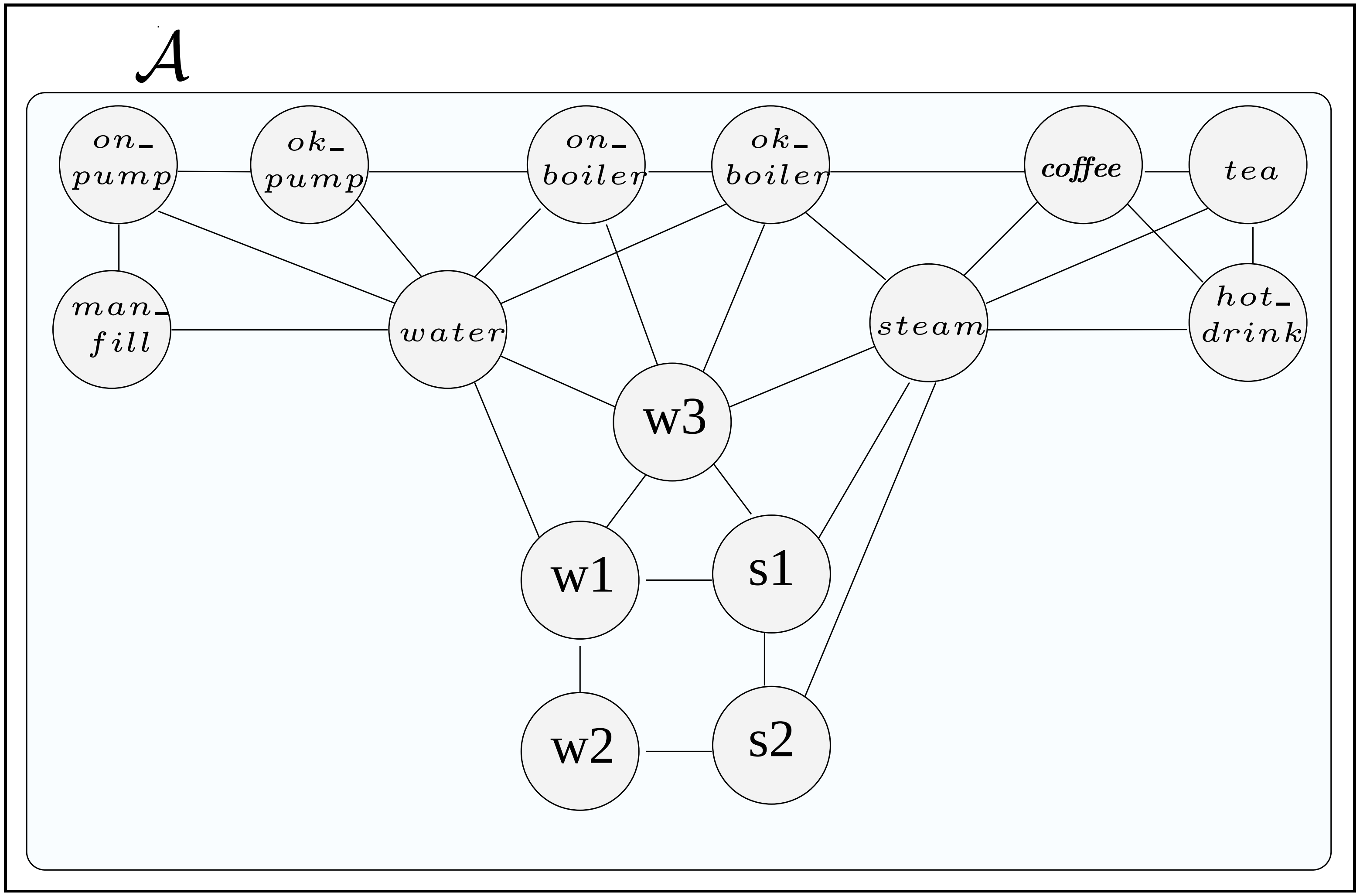
$\mathcal{A}$'s theory

Automated Partitioning using TRIANG.



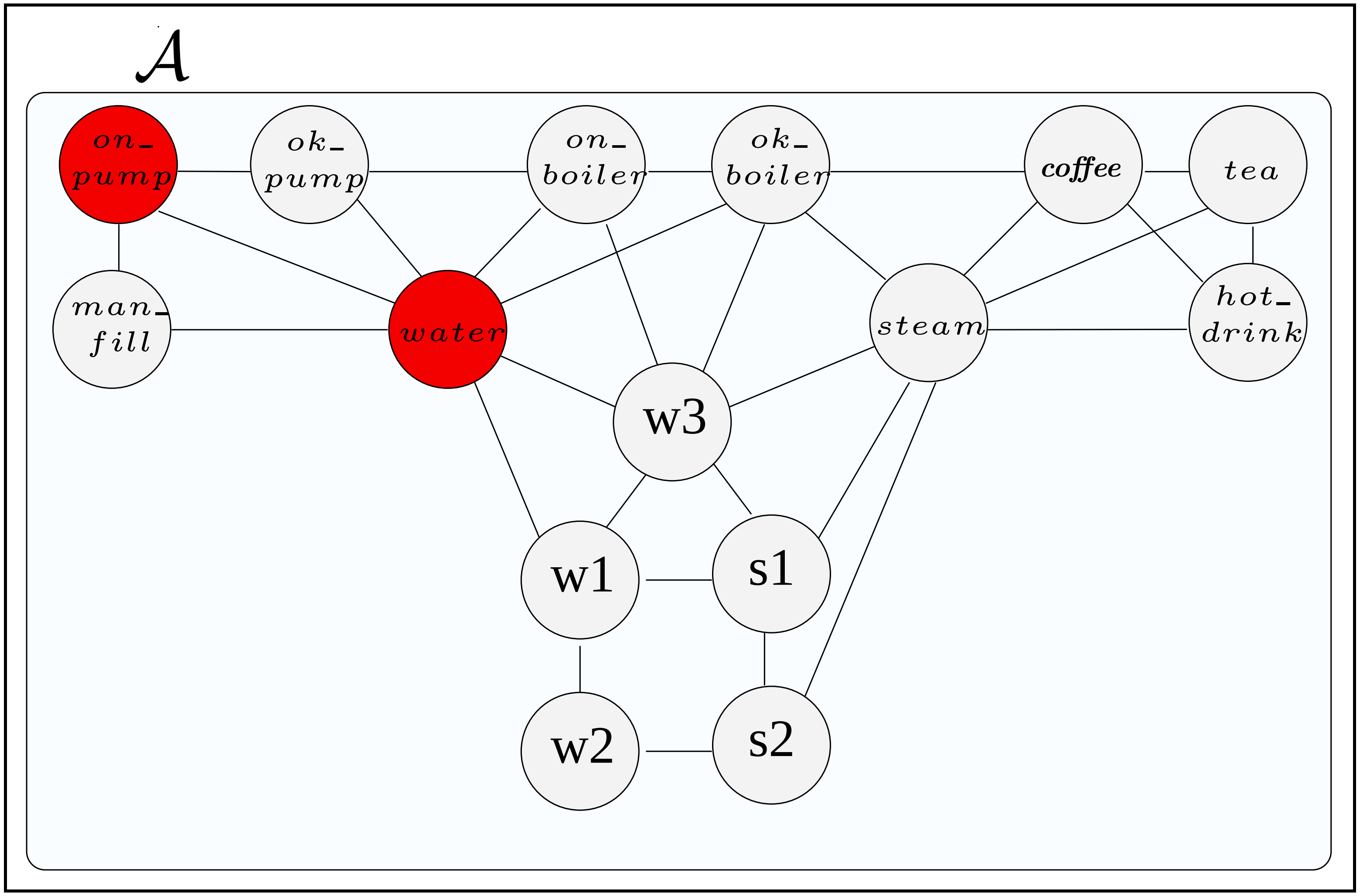
$\mathcal{A}$'s symbols graph

Automated Partitioning using TRIANG.



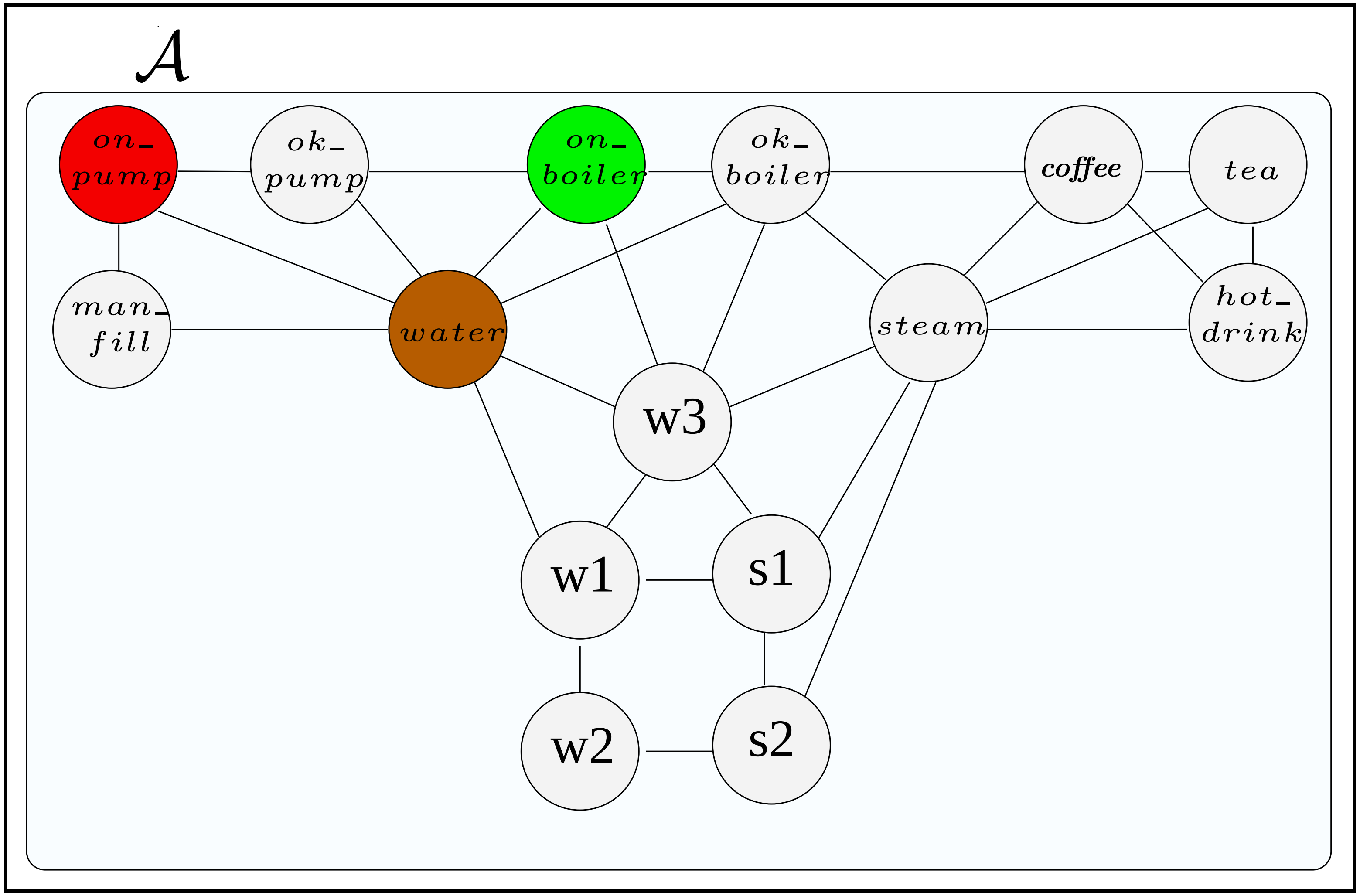
$\mathcal{A}$'s symbols graph (axioms/symbols appended).

Automated Partitioning using TRIANG.



Using a *balanced separator* of previous separators

Automated Partitioning using TRIANG.



Using a *balanced separator* of previous separators

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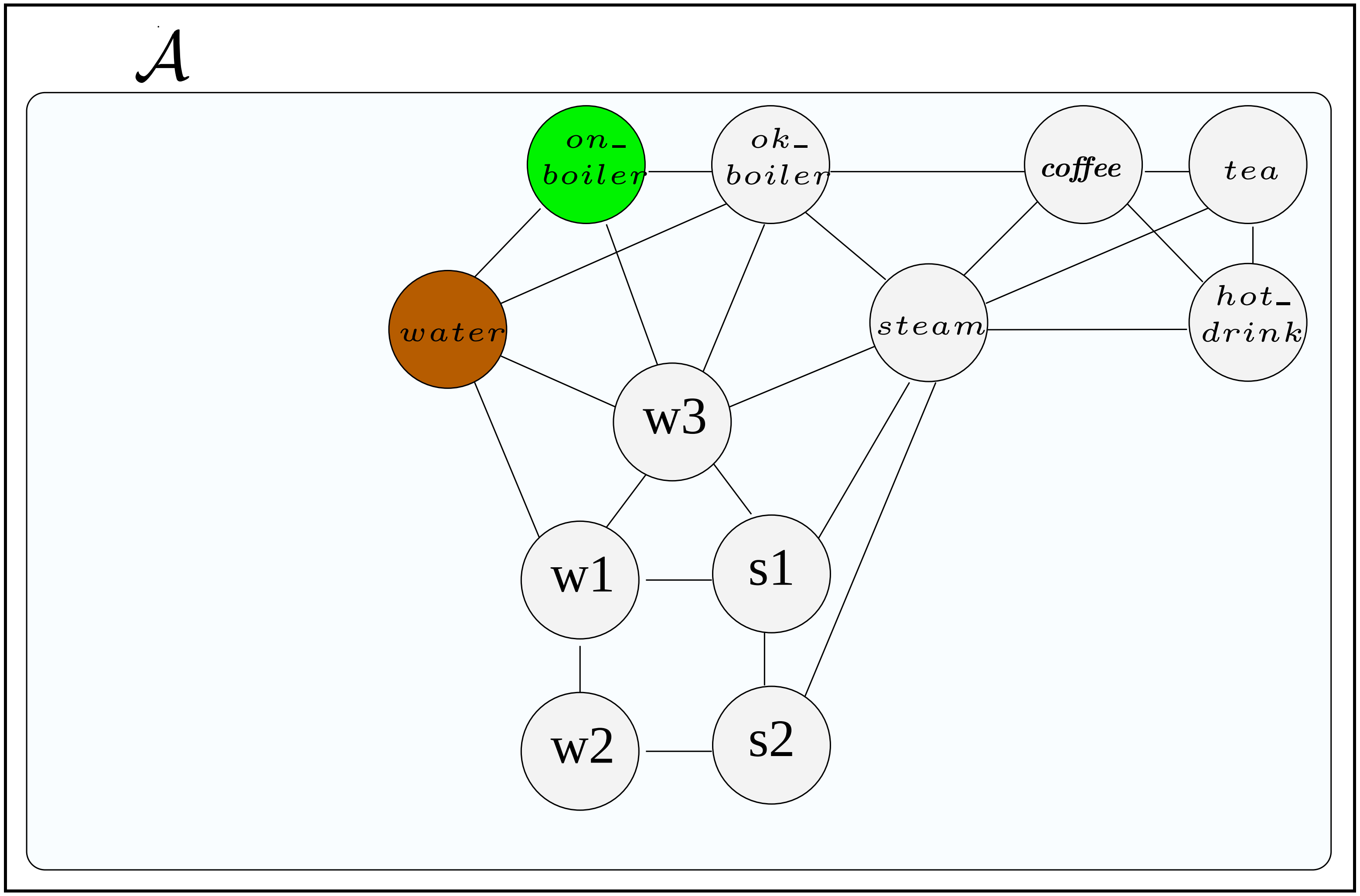

Using a *balanced separator* of previous separators

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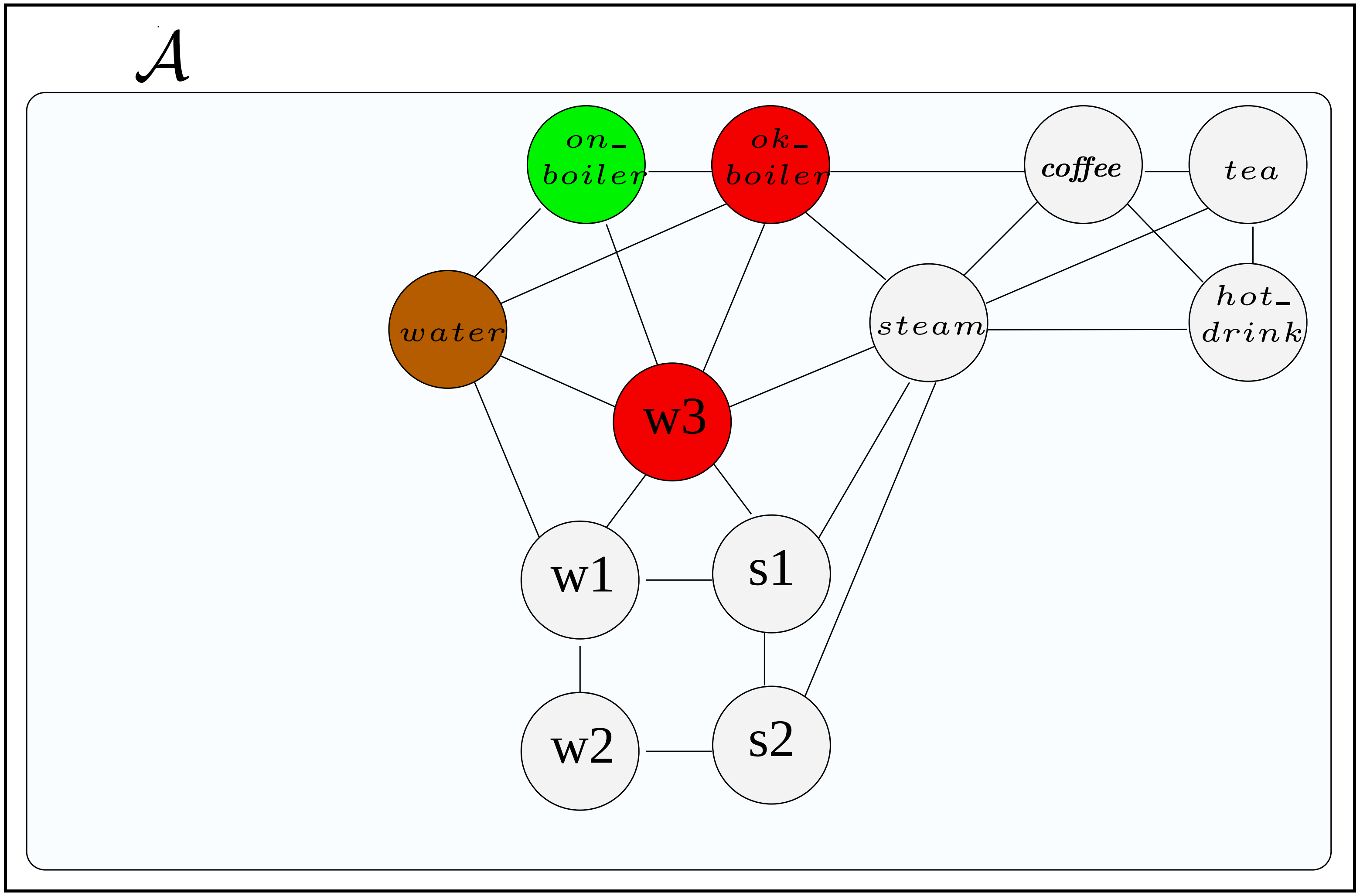

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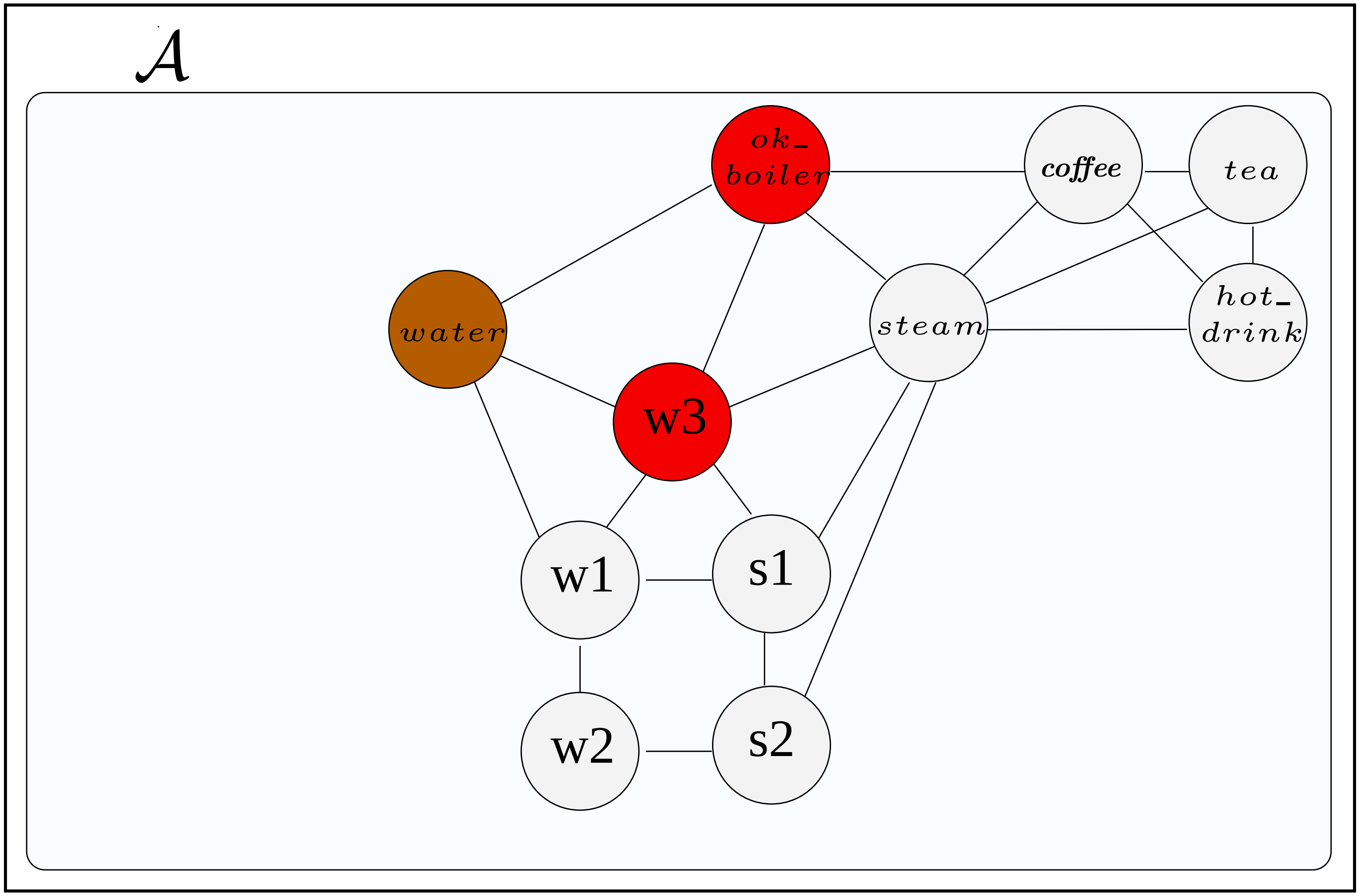

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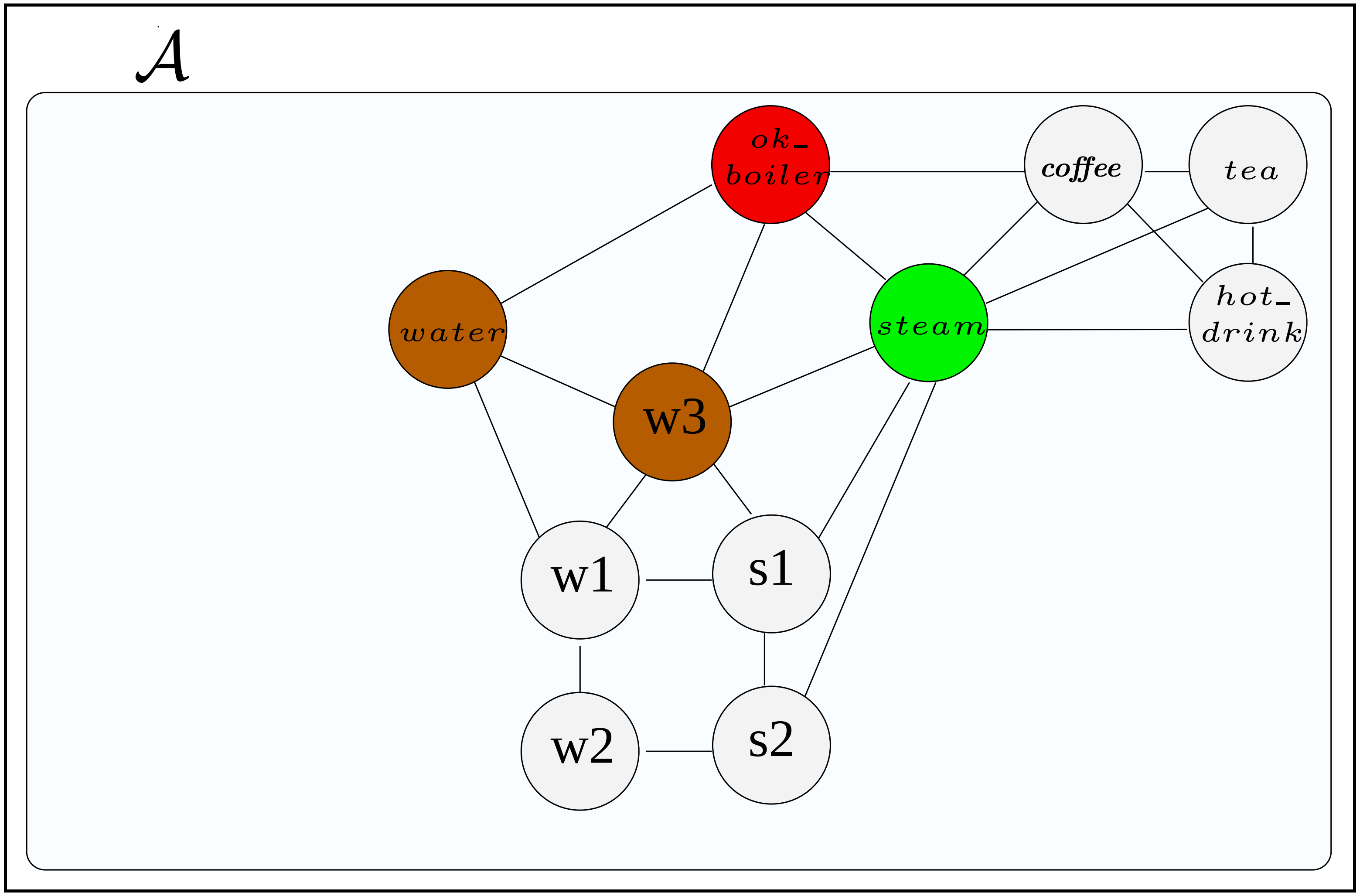
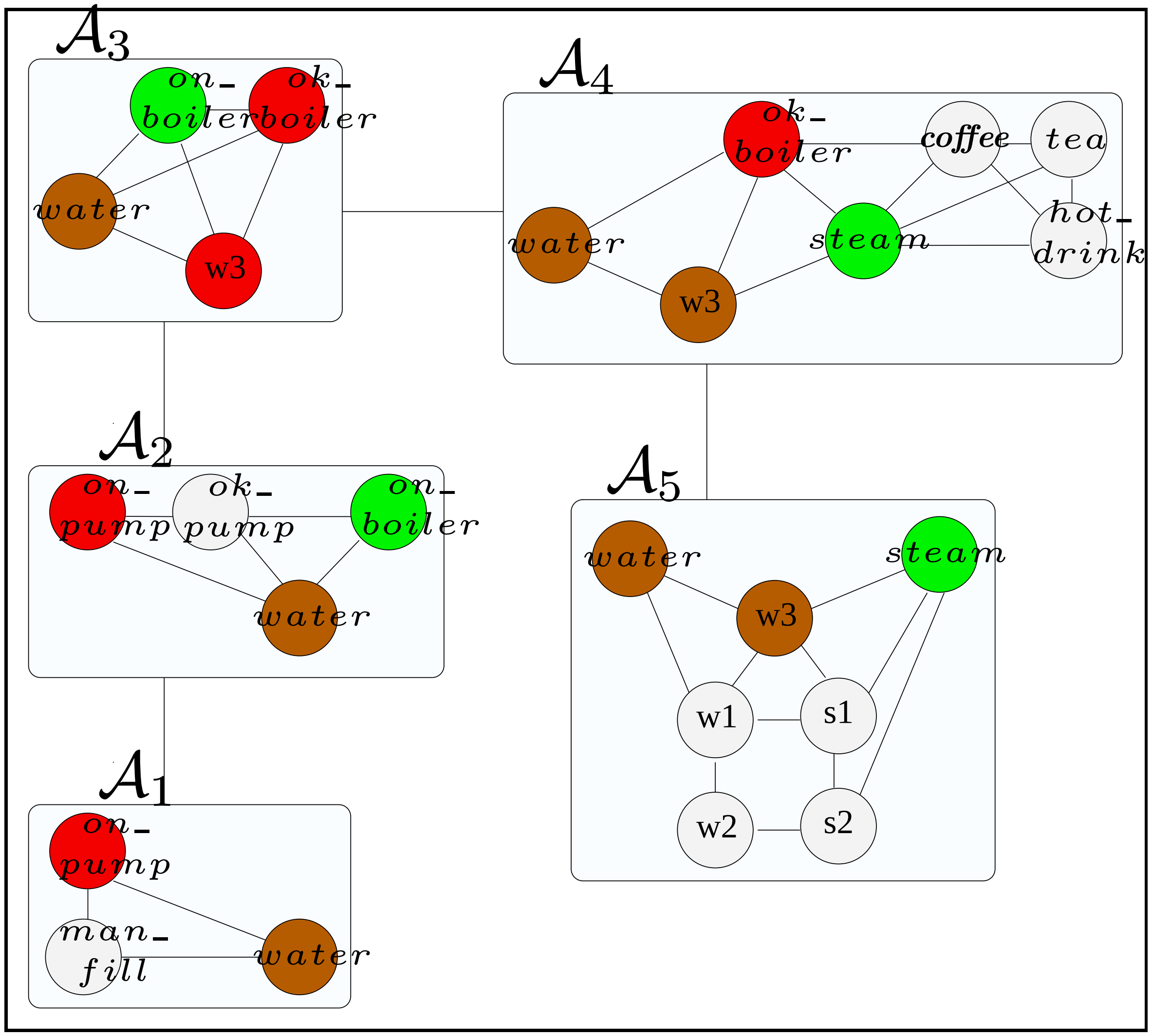
Automated Partitioning using TRIANG.



Using a *balanced separator* of previous separators

Automated Partitioning using TRIANG.



Overall decomposition graph.

Analysis of TRIANGULATE

- **TRIANGULATE is guaranteed to find a factor-3 approximation to the optimal.**
- **It takes time $O(2^{2k}k^{\frac{5}{2}}|V|^2)$ (k is the size of the largest partition in the optimal decomposition).**

Using TRIANGULATE on (some of) Cyc's Spatial Axioms

223 **axioms over** 142 **symbols.**

Time: 3.5 minutes

Largest partition: 21 **symbols.**

Largest link: 20 **symbols.**

Using TRIANGULATE on (some of) SRI's HPKB Axioms

Original Axioms

688 **axioms with** 570 **symbols.**

Time = 13 hours;

Largest partition with 70 **symbols.**

CNF axioms

1199 **axioms with** 446 **symbols.**

Time = 2.5 hours;

Largest partition with 58 **symbols.**

Summary

Motivating Problems:

- **Exploiting structure inherent in a set of logical axioms for efficient reasoning.**

Contribution:

- **Showed that commonsense knowledge bases are decomposable.**
- **We provide decomposition algorithms for FOL theories.**
- **Good Decomposition of theories provide computational advantages for theorem provers.**

Current & Future Work

- **Examine other decomposition algorithms.**
- **Analyze the decompositions of the HPKB axioms and Cyc's axioms and compare them to human-made decompositions.**
- **Experimental analysis of partition-based reasoning.**
- **Open directions: equality, types, logics, planning, prime-implicates.**

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Extensions

- **Relaxing completeness for consequence finding: $\mathcal{L}$-generation completeness**
- **Exploiting polarity using Lyndon's interpolation theorem**
- **Queries from multiple partitions**
- **Special treatment for first-order resolution.**

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Related Work

- **Bayes Nets: e.g., (Pearl 1988), (Becker & Geiger 1996).**
- **CSPs: e.g., (Dechter & Pearl 1988), (Gottlob et al 1999).**
- **Join Trees: e.g., (Shoikhet & Geiger 1997), (Kloks 1994).**
- **SAT: e.g., (Selman & Kautz 1993), (Dechter & Rish 1994), (Park & VanGelder 1996).**
- **Parallel Thm Proving: e.g., (Bonacina 1994), (Cowen & Wyatt 1993), (Suttner 1997).**
- **Combining Logical Systems: e.g., (Nelsson & Oppen 1979), (Shostak 1984), (Baader & Schulz**

1992).

- **Partitioned Representations: e.g., (McCarthy & Buvac 1998), (Giunchiglia & Ghidini 1998), (Amir 2000).**

Effectiveness of MP

Currently available measures for effectiveness of proof procedures:

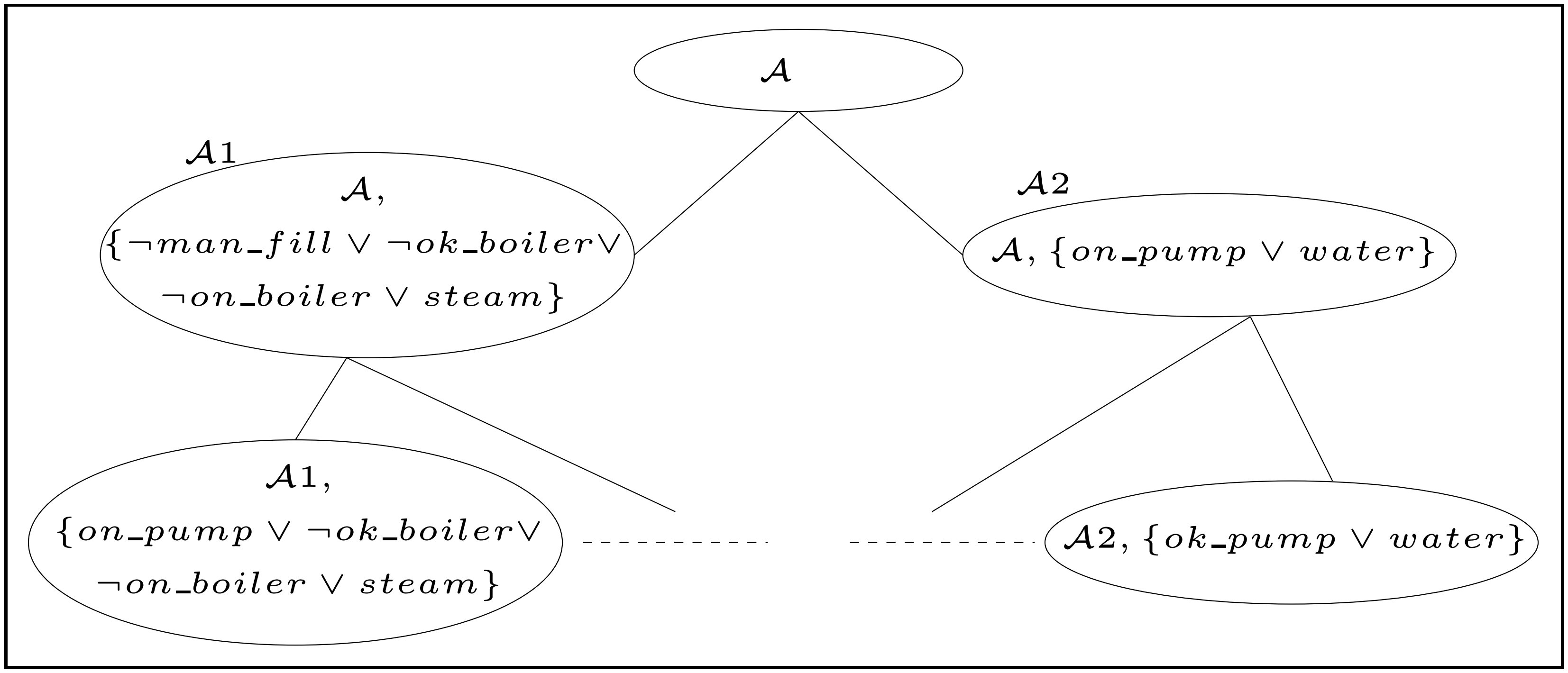
- **Search-space size**
- **Proof length**

Theorem 2 ***Resolution [w/restriction strategies] applied using MP has:***

- ***restricted search space, depending on partitions sizes and links sizes.***
- ***small interpolants and short proofs, depending on links sizes.***

Effectiveness of MP: Search Space

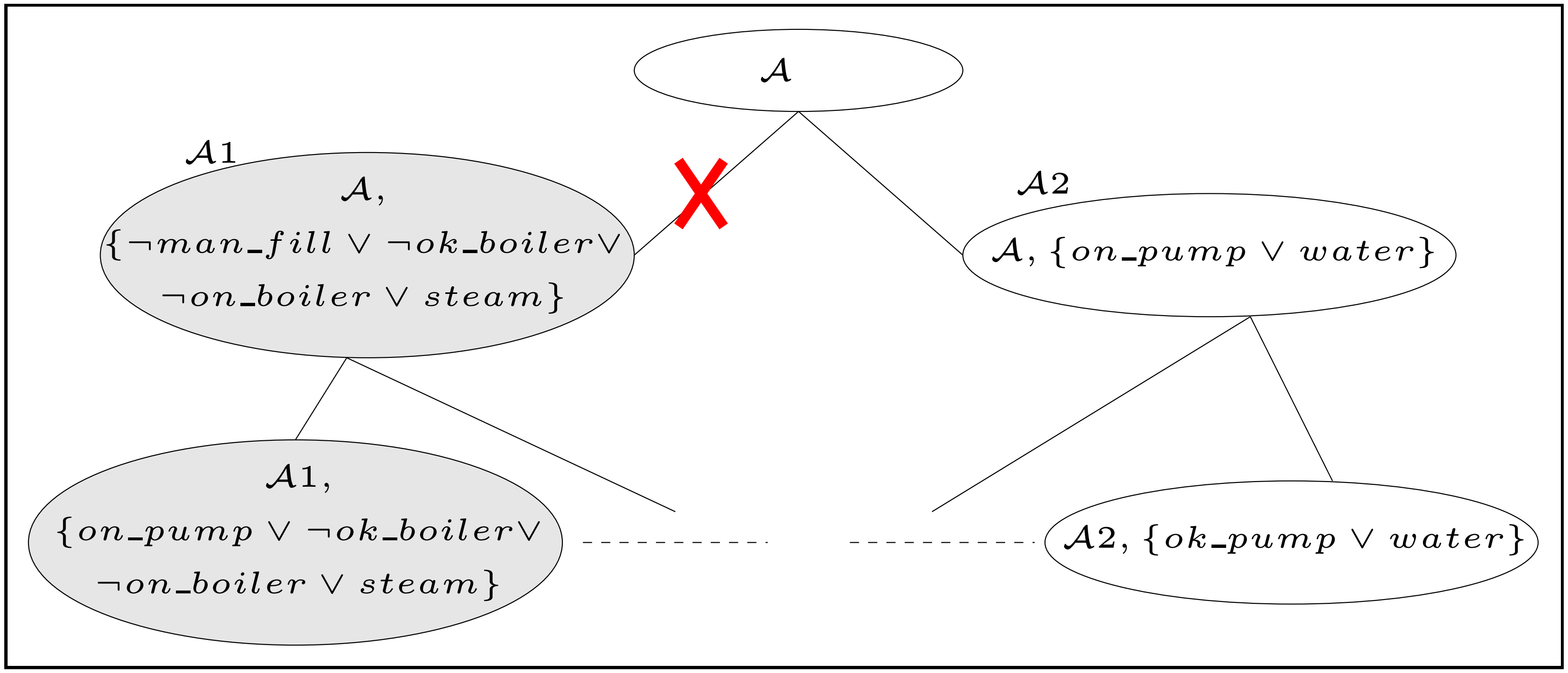
- **In resolution-based inference, the search space using MP is restricted.**



Linear Resolution

Effectiveness of MP: Search Space

- **In resolution-based inference, the search space using MP is restricted.**



Linear Resolution restricted with MP

Effectiveness of MP: Proof Length

Propositional proof lengths with/without MP:

- **Longest proof: The longest non-MP proof is as long or longer than the longest MP proof.**
- **Shortest proof:**
  - **The shortest non-MP proof can be shorter than the shortest MP proof.**
  - **Evidence: The difference in length between the shortest MP proof and shortest non-MP proof is polinomially bounded, if the size of links is fixed.**
  - **Evidence: The difference is exponentially**

bounded in the size of the links.