Automatic Population of Geographic Databases

GENERIC SENSOR MODEL

Lynn H. Quam

SRI International

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Goals of the Generic Sensor Model

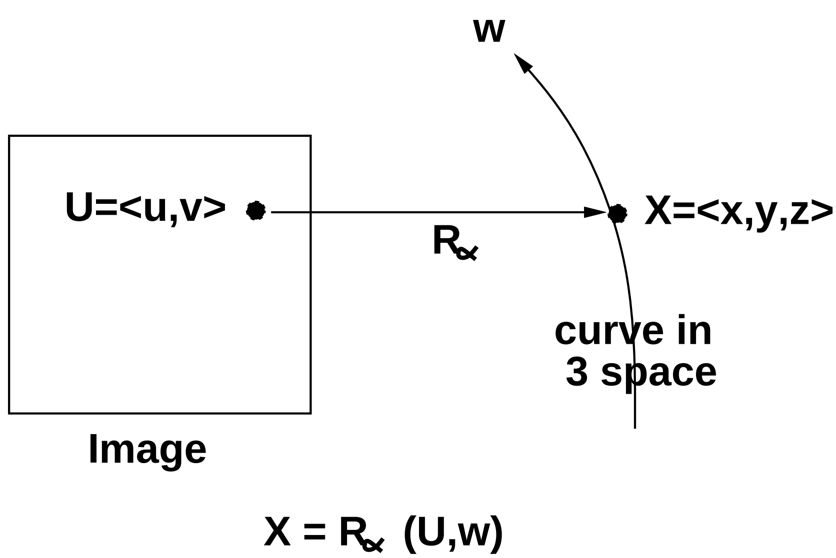
Specify a sensor model API suitable for modeling a large variety of imaging sensor geometries used in cartographic, GIS, and IMINT applications.

- Allows COTS software to support classified sensors with a totally unclassified sensor model implementation.
- Avoids proliferation of data formats describing the sensor parameters.
- New sensors can be added without changes to the API.
- Facilitates the transfer of IU algorithms developed in unclassified environments.
- Isolates applications from details of the sensors. Features unique to frame cameras are not provided.

Sensor Model Geometry

- World Coordinates: $X =< x, y, z >$
- Sensor Coordinates: $U =< u, v >$
- Rigorous Sensor Function: $X = R_\beta(U, w)$

  R is assumed to be differentiable with respect to u, v, w and components of β.



Mathematical Properties of the Sensor Model Function

The physical sensor is assumed to be rigorously modeled by the function $X = R(U, w)$, which for each pixel U in the image defines a parametric curve in the 3d world.

- $R(U)$ generates a parametric curve in 3-space corresponding to image point U.
- $R(U, w)$ is assumed to be continuous and differentiable with respect to u, v, and w. The partial derivitives if R with respect to u, v, and w may be computed either analytically or numerically.
- The **domain of** R is the set of points U covered by the image.
- The **range of** R is the 3-space volume defined by the set of all curves $R(U)$ for U within the **domain of** R
- The **bounded range of** R is the intersection of **range of** R with an application determined volume of 3-space. For example, the subset of 3-space with z between -300 meters and 1200 meters.
- R is assumed to be uniquely invertable within its **bounded range**. $P(X) = R^{-1}(X)$ projects world points into image points. The domain of P is the range of R. The range of P is the domain of R.

API Components

- Geometric Operations
  - Efficient implementation of $P(X)$.
  - Jacobi matrix $J_P(X)$ of P with respect to x, y, z at a point X.
  - $P^{-1}(U, w)$ inverse of P using Newton Raphson.
  - Ray-Surface intersection:
    Solve $S(P^{-1}(U, w) = 0$ for the free parameter w.
  - Multi sensor ray intersection:
- Statistical Operations - Covariance propagation.
  - ΣU for $U = P_\alpha(X)$ given $\Sigma\alpha$. Discussed later.
  - ΣU for $U = P(X)$ given ΣX.
  - ΣX for $X = P^{-1}(U)$ given ΣU.
  - ΣX for X computed from multi sensor ray intersection of rays.
  - ΣX for X computed from ray-surface intersection.
- Misc Operations
  - Input and Output of Sensor models.

API Reference Implementation

- A prototype of the reference implementation will be developed within RCDE.
- The reference implementation will be delivered as a C library.
- The projection function will be imnplemented by 3rd order rational polynomial functions **(RPF)** of the form specified in 200EAA.

  $$u = \frac{P_u(x,y,z)}{Q_u(x,y,z)} \qquad v = \frac{P_v(x,y,z)}{Q_v(x,y,z)}$$

  where numerator polynomials contain 20 terms and denominator polynomials contain 19 terms. There are an additional 10 normalization parameters.
- The **RPF** reference implementation directly computes $P(X)$ rather than $P^{-1}(U)$. This is critical to performance and conforms to 200EAA.
- For performance reasons, the 3-space coordinate system is Cartesian rather than geographic $< lat, long, elevation >$. The Cartesian coordinate system may be either geocentric coordinates $< gx, gy, gz >$ (WGS-84) or local vertical coordinates. This differs from 200EAA.
- We propose a format for storing **RPF** parameters that is different from that specified in 200EAA.

Current Status

The Radius Common Development Environment (RCDE) has supported a subset of generic sensor model API since about 1994.

- Sensor projection is approximated by the Fast Block Interpolation Projection **(FBIP)**.
- FBIP camera model files consist of tabulations of ground positions in 3 planes of constant world z corresponding to points on a regular grid in the image. These correspondences are generated using the RULER camera models and ESD data from DIDOP or NITF image headers.
- Partial derivitives are computed numerically.
- API geometric operations are supported. Speed is adequate for interactive manipulation of site models, updating multiple views thru FBIP projections.
- API covariance propagation is not well supported. The necessary covariance matrices are generated, but API is not defined.
- Six-parameter, rigid bundle adjustment is provided.

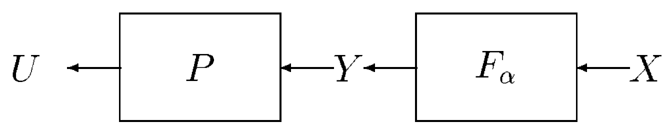
Schedule for Reference Implementation

	RCDE	C language
Basic Geometry &I/O	now	July 1, 97
Statistics - 200EAA-like model	July 1, 97	July 1, 97
Statistics - 6 parameter	July 1, 97	July 1, 97
Statistics - more parameters	(1)	(1)

(1) = Delivery depends on results of experiments and agreement on utility.

Generic Sensor Bundle Adjustment

- Adjustment Function F_α.
  Define $P_\alpha(X) = P(F_\alpha(X))$.
  - F is an invertable transformation from 3-space to 3-space.
  - F has a set of parameters α.
  - F is differentiable with respect to components of X and α.

  Example: Rigid motion.

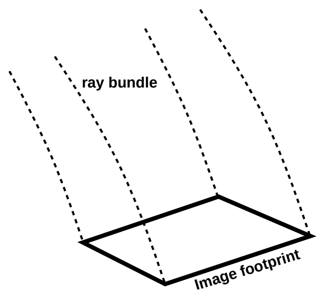
  $$F_{\omega,\phi,\kappa,x_0,y_0,z_0}(X) = R_{\omega,\phi,\kappa} \cdot X + < x_0, y_0, z_0 >.$$

  where R is an orthonormal rotation matrix defined by the parameters ω, ϕ, κ.

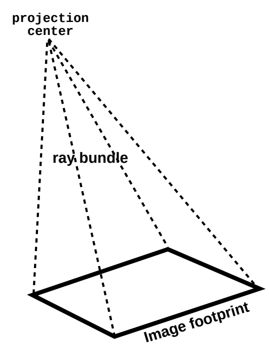
- We assume that the function $P(X)$ provides a good starting approximation to the actual sensor projection. Thus the a priori estimates to all components of α are zero.

- Perform "ordinary" least squares adjustment using $P_\alpha(X)$ in the colinearity equations. A priori covariances for $\Sigma\alpha$ my be supplied as discussed in the next section.

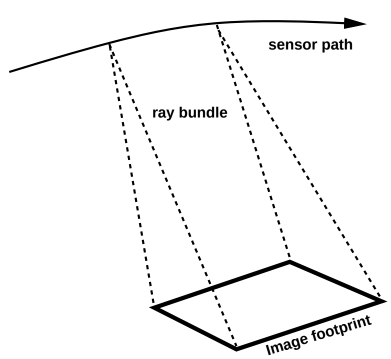
Ray Bundles of Various Sensors



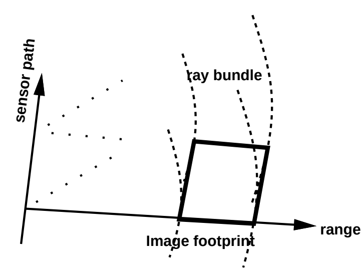
General Case



Frame Camera



Dynamic Sensor



SAR Sensor

Conventional Covariance Propagation from Rigorous to Generic Sensor Model

Equations for covariance propagation:

> $P_1(\alpha_1, X)$ is the rigorous projection function.
> $P_2(\alpha_2, X)$ is the generic projection function.
> B_j is the matrix consisting of a column of Jacobian matrices
> of $P_j(X_i)$ where X_i is a set world points.

$$\Sigma L = B_1 \Sigma\alpha_1 B_1^T$$

Solve for $\Sigma\alpha_2$ such that

$$\Sigma L = B_2 \Sigma\alpha_2 B_2^T$$

The "conventional" solution to this matrix equation is:

$$\Sigma\alpha_2 = (B_2^T (B_1 \Sigma\alpha_1 B_1^T)^{-1} B_2)^{-1}$$

Unfortunately, if we attempt to use this formulation to propagate covariances $\Sigma\alpha_1$ thru P_1 and P_2^{-1} we encounter singular matrices in either or both of the matrix inversions.

Improved Covariance Propagation from Rigorous to Generic Sensor Model

Due to differences in the number of parameters of P_1 and P_2, and the number of points X_i we have matrices with different ranks, and usually either or both of the matrix inversions in the above formulation involves a singular matrix. We have found that the following formulation provides significantly better numerical behavior:

Let B^{-S} be the result of matrix "pseudo inversion" using singular value decomposition (**SVD**). Then:

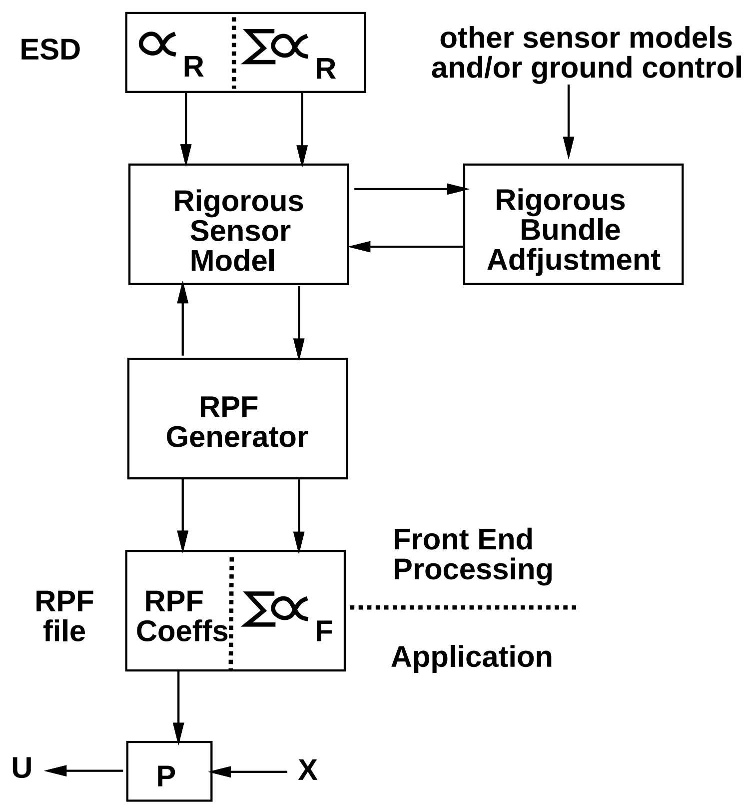
$$\Sigma\alpha_2 = (B_2^{-S}B_1)\Sigma\alpha_1(B_2^{-S}B_1)^T$$

We believe that we can prove that this solution for $\Sigma\alpha_2$ is the least squares minimum solution to the equation:

$$|B_1\Sigma\alpha_1 B_1^T - B_2\Sigma\alpha_2 B_2^T|^2 = min$$

where $|M|^2$ is the sum of squares of the elements of matrix M.

RPF Coefficient Generation Front End



Application Bundle Adjustment