Digital Stereoimage Transformation Procedures*

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January 1991

1 Introduction

The U.S. Army Topographic Engineering Center (USATEC) has accumulated various digital stereomodels of photographic images for use in projects involving automatic feature extraction, elevation data extraction, etc. The stereomodels have been triangulated to known ground control and, consequently, the necessary data exist for the transformation of stereoimage coordinates into X, Y and Z-ground coordinates in an absolute ground coordinate system.

In the interest of making these images and data sets available to other users in a consistent manner, USATEC has selected a format by which the data can be reported and has outlined procedures for using the data. The data will be tabulated in "USATEC Image Transformation Parameters" reports. The procedures for using the data are described in the following sections.

2 Procedures

Coordinates of digital images are generally measured on some sort of image display equipment with a specific cursor origin and axes definition. Since the origin and direction of axes vary from system to system, it is not practical to outline transformation procedures that begin with "screen" coordinates of a measured image point. The procedures given herein start with "file" coordinates which are the column (x) and row (y) coordinates of an image as it appears in a digital file. It will be the responsibility of the user to provide the means for converting screen coordinates to file values. Where appropriate, the data values that appear in the "USATEC Image Transformation Parameters" reports are predicated on a file structure that starts with element $(0,0)$. Columns (x) increase from left to right and rows (y) increase from top to bottom.

Step 1. File-to-Image Transformation.

In these procedures, a distinction is made between a file and an image in order to account for the possibility that a subset of the original image may be used and that the subset may be rotated or reflected relative to the original image. The first step is, therefore, to convert file values to image values as follows:

$$\begin{aligned} x_i &= a \cdot x_f + b \cdot y_f + c & \text{(columns)} \\ y_i &= d \cdot x_f + e \cdot y_f + f & \text{(rows)} \end{aligned}$$

Where:

*Corrected by A. Heller, SRI International, July 1998.

x_i, y_i ..... col (x) and row (y) coordinates (pixels) of the image point in the original image.

$x_f \cdot y_f$ ..... col and row coordinates (pixels) of the image point in the displayed file.

a, b, d, e ..... ones and zeros with the appropriate signs for rotation and reflections of the axes.

c, f ..... translation of the $(0,0)$ element of the file relative to the $(0,0)$ element of the image.

Step 2. Image-to-Scanner Transformation.

A rigorous image-to-ground conversion can be achieved with triangulation data only if the "interior orientation" of the original photographs can be recaptured. Interior orientation is the process by which measured photograph coordinates are transformed to a coordinate system as defined by the calibrated fiducials of the taking camera. In the process of digitizing the photographs, it is necessary for interior orientation purposes (1) to measure the scanner coordinates of the fiducial marks, (2) to measure the scanner coordinates of the starting point of the digitized area and (3) to record the sampling size between pixels. The image-to-scanner transformation can be achieved as follows:

$$
\begin{aligned}
x_s &= ((a \cdot x_i + b \cdot y_i) \cdot s + x_o)/1000. \\
y_s &= ((c \cdot x_i + d \cdot y_i) \cdot s + y_o)/1000.
\end{aligned}
$$

Where:

a, b, c, d ..... signed ones and zeros to indicate direction of scan.

x_s, y_s ..... scanner coordinates (mm) of the image point.

x_o, y_o ..... scanner coordinates (microns) of the starting point of the digitized area.

s ..... spacing between pixels (microns).

Step 3. Scanner-to-Fiducial Transformation.

As a separate processing step, a computation is performed to determine the parameters necessary to fit the scanner coordinates of the fiducial points to the values given in the camera calibration report. These parameters are used subsequently to convert other scanner coordinates of image points into the fiducial coordinate system as follows:

$$
\begin{aligned}
x_{\text{fid}} &= a \cdot x_s + b \cdot y_s + c \cdot x_s \cdot y_s + d \\
y_{\text{fid}} &= e \cdot x_s + f \cdot y_s + g \cdot x_s \cdot y_s + h
\end{aligned}
$$

Where:

$x_{\text{fid}}, y_{\text{fid}}$ ..... coordinates (mm) of image point in the fiducial coordinate system.

a, b, c, e, f, g ..... rotation and scaling parameters.

d, h ..... translational parameters.

Step 4. Lens Distortion Corrections.

Corrections are made to the x and y values in the fiducial coordinate system in order to account for distortions in the lens of the taking camera. The method assumed in the "USATEC Image Transformation Parameters" report involve look-up procedures. The corrections are applied as follows:

$$
\begin{aligned}
R &= (x_{\text{fid}}^2 + y_{\text{fid}}^2)^{1/2} \\
\text{CON} &= (R - D)/R \\
x_1 &= \text{CON} \cdot x_{\text{fid}} \\
y_1 &= \text{CON} \cdot y_{\text{fid}}
\end{aligned}
$$

Where:

x_1 ,y_1	.....	coordinates (mm) of image point corrected.
CON	.....	proportionality constant.
R	.....	radius on image to point to be corrected
D	.....	lens distortion (mm) at radius R (mm).

Step 5. Atmospheric Refraction Correction.

Atmospheric refraction corrections are customarily applied as part of the triangulation process. They should also be applied when the triangulation data are used for other applications. However, compared to the size of a pixel, the corrections are usually very small and can be ignored. If atmospheric refraction was corrected in the triangulation process, the appropriate parameter will also be tabulated in the "USATEC Image Transformation Parameters" report. Schut's method as described in "photogrammetric defraction," *Photogrammetric Engineering*, is assumed to be used for this purpose. The resulting coordinates will be x_p and y_p.

Step 6. Coordinate Rectification.

The corrected x_p and y_p coordinates for both images can be used in conjunction with the orientation data obtained from the triangulation process to compute ground coordinates. The first step is to rotate the photo-coordinates of the image points into a set of axes that are parallel to those of the ground coordinate system used in the triangulation process. This involves the transpose of the ground-to-photo rotation matrix, ORM, as follows:

$$
\begin{bmatrix} x_{r1} \\ y_{r1} \\ z_{r1} \end{bmatrix} = [\text{ORM}_1]^{\mathsf{T}} \cdot \begin{bmatrix} x_{p1} \\ y_{p1} \\ -f_1 \end{bmatrix} \qquad \begin{bmatrix} x_{r2} \\ y_{r2} \\ z_{r2} \end{bmatrix} = [\text{ORM}_2]^{\mathsf{T}} \cdot \begin{bmatrix} x_{p2} \\ y_{p2} \\ -f_2 \end{bmatrix}
$$

Where:

x_p, y_p, f	.....	photocoordinates (mm) of the image point and the calibrated focal length (mm) of the camera.
x_r ,y_r, z_r	.....	image coordinates (mm) in a system parallel.
ORM	.....	the ground-to-photo rotation matrix for each photo as given by the triangulation process. ORM transposed is actually used.

The ORM matrix for each photograph can be defined in terms of three sequential rotations of the ground coordinate axes into the coordinate system of the photograph. The rotations are omega, phi and kappa. Omega is a rotation about the *x*-axis, phi is about the once-rotated *y*-axis and kappa is about the twice-rotated *z*-axis. The angles are positive clockwise as viewed from the positive ends of the axes and toward their origin.

$$
[\text{ORM}] = [\kappa]\,[\varphi]\,[\omega]
$$

Step 7. Transform to Ground Coordinates

The rectified photocoordinates can now be used along with the local rectangular coordinates of the camera stations to compute the ground coordinates XG, YG and ZG of the measured stereoimage point.

photo 1	photo 2
$\frac{x_{r1}}{z_{r1}} = \frac{XG - XC_1}{ZG - ZC_1}$	$\frac{x_{r2}}{z_{r2}} = \frac{XG - XC_2}{ZG - ZC_2}$
$\frac{y_{r1}}{z_{r1}} = \frac{YG - YC_1}{ZG - ZC_1}$	$\frac{y_{r2}}{z_{r2}} = \frac{YG - YC_2}{ZG - ZC_2}$

Where:

XC, YC, ZC ..... local rectangular ground coordinates (m) of the camera stations.
XG, YG, ZG ..... local rectangular ground coordinates (m) of the stereoimage point.
x_r, y_r, z_r ..... rectified photocoordinates (mm).

If the expressions for x_{r1} and x_{r2} (or y_{r1} and y_{r2}) are combined to eliminate XG (or YG), the resulting expression can be used to compute ZG. The value of ZG can be substituted back into the equations to compute XG and YG. Combine the equations in the x-direction. Combine the y-equations, otherwise.

Step 8. Local-to-Geographics and UTM Transformations.

The XG, YG and ZG ground coordinates are computed in a local rectangular coordinate system. If the geographic position and height of the origin of the local system are known, the local values can be converted to geographic and Universal Transverse Mercator (UTM) coordinates using a rigorous coordinate conversion routine. Alternately, locals can be transformed using an approximate solution which is both fast and accurate over limited geographic areas. The approximate method uses a 6-term polynomial to map changes (from the origin) in local XG and YG coordinates into changes in geographics or UTMs. The local ZG coordinate value influences the changes in geographics and UTMs and must be accounted for in the polynomials. The affect of ZG is considered by adjusting the XG and YG values before they are used in the polynomial transformation.

$$
\begin{aligned}
ec &= ((XG = XG_o)^2 + (YG - YG_o)^2)/2R_m \\
h &= ZG + ZG_o + ec \\
\mathrm{dz} &= (1.0 - h/R_m) \\
xm &= (XG - XG_o) \cdot \mathrm{dz} \\
ym &= (YG - YG_o) \cdot \mathrm{dz}
\end{aligned}
$$

Where:

XG, YG, ZG ..... local coordinates (m) of any ground point.
YG_o, YG_o, ZG_o ..... local coordinates (m) of origin. XG and YG of the origin are always zero.
ec ..... earth curvature correction (m) to ZG.
h ..... height of point (m) above sea level.
R_m ..... mean radius of curvature (m) of the earth at the geographic origin.
dz ..... scale factor to reduce XG and YG values at a ZG-height to values at zero-height.
xm, ym ..... modified XG and YG values (m) for use in the transformation polynomials.

The xm and ym coordinates can now be used in the polynomials to compute corresponding changes in geographics or UTMs relative to the origin of the local system. If geographic changes are computed, the dx and dy terms are in units of arc second. If UTM changes are desired, the units of dx and dy are meters. The dx polynomial transforms local XG coordinates into changes in longitude or Easting, depending on the transformation that is to be performed. The dy expression transforms local YG values into latitude or Northing changes. Implied here is that the local XG axis is always oriented easterly and YG points northerly.

$$\begin{aligned} \mathrm{d}x &= a_0 + a_1 \cdot xm + a_2 \cdot ym + a_3 \cdot xm^2 + a_4 \cdot ym^2 + a_5 \cdot xm \cdot ym \\ \mathrm{d}y &= b_0 + b_1 \cdot xm + b_2 \cdot ym + b_3 \cdot xm^2 + b_4 \cdot ym^2 + b_5 \cdot xm \cdot ym \end{aligned}$$

The absolute height of the local ground point and the geographic and UTMs coordinates are computed as follows:

$$\begin{aligned} \lambda &= \lambda_0 + \mathrm{d}x & E &= E_o + \mathrm{d}x \\ \varphi &= \varphi_0 + \mathrm{d}y & N &= N_o + \mathrm{d}y \\ h &= ZG + ZG_o + ec & h &= ZG + ZG_o + ec \end{aligned}$$

Where:

λ_o, φ_o ..... longitude and latitude coordinates (secs) of the local origin.
E_o, N_o ..... Easting and Northing coordinates (m) of the local origin.