Temporal Constraint Reasoning with Preferences and Probabilities

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Abstract

In an uncertain world, a rational planning agent must simultaneously reason with uncertainty about expected outcomes of actions and preferences for those outcomes. This work focuses on systematically exploring the interactions between preferences for the durations of events, and uncertainty, expressed as probability distributions about when certain events will occur. We expand previous work by representing events and durations that are not under the control of the agent, as well as quantitative beliefs about when those events are likely to occur. Two reasoning problems are introduced and methods for solving them proposed. First, given a desired overall preference level, compute the likelihood that a plan exists that meets or exceeds the specified degree of preference. Second, given an initial set of beliefs about durations of events, as well as preferences for times, infer a revised set of preferences that reflect those beliefs.

1 Introduction

Rational agents are capable of mentally exploring the interactions between what they believe and what they desire as outcomes of actions. More often than not, the value of the outcomes of actions cannot be described by a single attribute, but rather by attributes that combine to determine the overall value of the outcome [8]. Furthermore, the outcome of actions may not be known with certainty, as a result of the need to interact with the world.

Many practical planning or scheduling problems surround events that are not controlled by the planning agent. For example, Earth Science observation scheduling may involve assigning times for the remote sensing of an area of interest on the Earth either before, during, or after a fire has occurred within that area. The start and end of the fire are not known with certainty at planning time, but Earth Science models might be available to estimate a set of times when fires are likely to occur. In addition, the scientific utility of an observation may vary based on when the observation is taken relative to the fire, resulting in preferences for temporal orderings and durations between planned events and uncontrollable events [13]. As automated planning matures as a software technology, new techniques inspired by decision theory are being integrated to address the fact that plans are executed in the world, with varying degrees of value to the planner based on their outcomes [1]. A principled approach to scheduling problems such as the above is essential for a decision-theoretic temporal planner that takes into account preferences when determining plan quality.

The goal in this paper is to devise systematic methods for exploring the interactions between temporal preferences and uncertainties. We introduce a framework that generalizes the Simple Temporal Problem (STP) formulation [4], called the Simple Temporal Problem with Preferences and Probabilities, or STP⁺. One component of the generalization adds the capability to express preferences for times, following [9]. The other component allows for the designation of uncontrollable events and the associated probability space over times.

Besides defining the STP⁺ framework, the contribution of this paper is to describe solutions to two practical reasoning problems arising from the interactions between probabilities and preferences. We extend techniques previously used to solve temporal problems with preferences to identify solutions that are both globally preferred and highly probable.

Decision-theoretic planning is surveyed by [1]. Most approaches either extend classical planning techniques or employ Markov Decision Processes (e.g. [31]), in contrast to our constraint-based focus. Of work on temporal reasoning for planning, a characteristic example is [7], who, like us, consider exogenous events, but who focus on eliciting probabilities and qualitative preferences from a human expert.

In the constraints literature, preferences are commonly represented using semiring-based formulations, the approach we adopt. An alternative formulation for qualitative preferences is CP-nets [2]. Uncertainty has also been represented both qualitatively and quantitatively; probabilistic frameworks include that of [6], which we adopt, and its extensions.

Generic constraint-based frameworks that account for both preferences and uncertainty include [5]. Our work is distinguished by restricting attention to Simple Temporal constraints. Prior work in this line has considered STPs with preferences but no uncertainty [9]; and STPs with uncertainty constraints but no preferences [12; 17]. While [15] incorporate both aspects, that work considers only qualitative uncertainty, that is, with implied uniform distributions.
2 Example: Earth Science Campaign Observation Scheduling

An Earth Science campaign is a systematic set of activities undertaken to meet a particular science objective. Here, we present a hypothetical campaign based on a science objective to test an emissions model predicting the aerosols released by wildfires. Data on several variables must be gathered in order to accomplish the analysis, and several remote sensors, such as those on the Landsat satellite, provide data products at various spatial resolutions relevant to these variables. Preferred times for acquiring Landsat data for vegetation type for a region of interest in the northern hemisphere would be the prior June or July in the same year that the fire burned, when forested land can most easily be spectrally distinguished from grassland. For mapping aerosol concentration, images coincident to burning must be obtained; the Terra and/or Aqua satellites have relevant instruments. For the burned area, data should be acquired after (though not too long after) the fire is out, while mapping vegetation moisture content, hyperspectral data from an EO-1 Hyperion instrument are relevant, and the most useful data would be that acquired just preceding the fire.

From this description, the inputs to a campaign planning problem potentially consist of the following characteristics:

- a set of temporal, spatial, and resource constraints on when and where images are to be taken;
- user preferences for when an observation should be taken; and
- temporal ordering constraints between planned events and uncontrollable, exogenous events such as fires.

A reasonable goal, given these inputs, is to generate a concise representation of the set of solutions (assignments of times and sensing resources) that are maximally preferred and reflect a set of initial beliefs about when exogenous events are likely to occur. The next section formulates a framework capable of describing the problem and generating this output.

3 Simple Temporal Problems with Preferences and Probabilities

A soft temporal constraint depicts restrictions on the distance between arbitrary pairs of distinct events, and a user-specified preference for a subset of those distances. In Khatib et al. [9], a soft temporal constraint between events $i$ and $j$ is defined as a pair $(I, f_{ij})$, where $I$ is a set of intervals $\{[a, b], a \leq b\}$ and $f_{ij}$ is a local preference function from $I$ to a set $A$ of admissible preference values.\(^1\) When $I$ is a single interval, a set of soft constraints defines a Simple Temporal Problem with Preferences (STPP), a generalization of a Simple Temporal Problem [4]. An STPP can be depicted as a pair $(V, C)$ where $V$ is a set of variables representing events or other time-points, and $C = \{\{a_{ij}, b_{ij}, f_{ij}\}\}$ is a set of soft constraints defined over $V$. An STPP, like an STP, can be organized as a network of variables representing events, and links labeled with constraint information.

Following recent approaches [6; 12; 17; 15], we extend the STPP framework to represent temporal uncertainty. First, we partition $V$ into two groups: the decision variables $V_d$ and the parameters $V_u$ representing uncontrollable events. We further distinguish between binary decision constraints ($C_d$), those which the agent executing the plan must satisfy, and uncertainty constraints ($C_u$), those which "nature" will satisfy.

An uncertainty (temporal) constraint depicts a duration as a continuous random variable. To ease the exposition, we assume that the uncertainty constraints are mutually independent;\(^2\) this allows the constraints in $C_u$ to be expressed in the form $\{[a_{ij}, b_{ij}, p_{ij}], \text{ where } p_{ij} : [a_{ij}, b_{ij}] \rightarrow [0, 1]\}$ is the probability density function over the designated interval. We call the framework $(V_d, V_u, C_d, C_u)$, where $C_d$ are soft constraints, a Simple Temporal Problem with Preferences and Probabilities, or STPP\(^3\).

Example 1 Earth Science Observation Problem. Inputs: Variables in $V_d$ standing for two controllable events consisting of taking an observation (Obs1, Obs2), and two uncontrollable events in $V_u$, the start and end of a fire ($FS, FE$) (for simplicity, observations are viewed as instantaneous), as shown in Figure 1. There is also an event TR representing the beginning of time. Soft constraints $f_1(t), f_2(t)$ in $C_d$ are associated with the durations between Obs1 and FS, and between Obs2 and FE, respectively. For example, $f_1(t)$ may express that there is no value for taking Obs1 after the start of the fire ($FS$), and a preference for times that are as close to $FS$ as possible. Similarly, $f_2(t)$ expresses a preference for Obs2 happening before $FE$ as close as possible, with a penalty if the observation is taken after the fire. Uncertainty constraints $p_1, p_2$ in $C_u$ are associated with random variables representing the start time and the duration of the fire. These constraints are based on Earth Science models about fires in the area of interest. For example, $p_1$ may express a normal distribution over the range of times.

\(^1\)For the purposes of this paper, we assume the values in $A$ are totally ordered, and that $A$ contains designated values for minimum and maximum preference.

\(^2\)For instance, imagine that the Earth Science planner maintains a Bayes network elsewhere to express the dependencies; each probability $p(t)$ is given implicitly by that network.
A solution to an STP\(^3\) is a set of assignments to \(V = V_d \cup V_u\) that satisfies all the constraints in \(C = C_d \cup C_u\). Given an STP\(^3\) \(P\), let \(\text{Sol}(P)\) be the set of all solutions to \(P\). An arbitrary solution \(s \in \text{Sol}(P)\) can be viewed as having two parts: \(s_d\), the set of values assigned to \(V_d\), and \(s_u\), the set of values assigned to \(V_u\).

Our goal is to develop efficient methods for generating a concise, graphical representation of subsets of \(\text{Sol}(P)\) corresponding to highly likely, globally preferred solutions. This STP-based graphical representation is called a flexible (temporal) plan. Many planning systems use an STP-based representation of the temporal aspects of their plans [16].

Following previous efforts, methods for flexible temporal planning under uncertainty can be distinguished based on assumptions about the strategy to be applied in executing the flexible plan. A static execution strategy assumes no access to the values of \(s_u\) during plan execution; by contrast a dynamic execution strategy is applied as plan execution proceeds and the values of \(s_u\) are observed over time [12; 15]. The results of this paper assume a static execution strategy; we defer discussions of planning for dynamic execution of STP\(^3\)s to future work.

**Component Solvers.** The solution methods described below are based on different decompositions of an STP\(^3\) into component sub-problems for which efficient solution methods exist. As a final preliminary, we fix some terminology and briefly summarize these sub-problem solution methods. Given an STP\(^3\), the underlying STPP is the problem that results when a constraint \([a, b], f_{XY}\) \(\in C_u\) is replaced by the STP component constraint \([a, b]\). The underlying Probabilistic STP is the problem that results when each soft constraint \([a, b], f_{XY}\) \(\in C_d\) is replaced by the STP component constraint \([a, b]\). The underlying STP replaces all constraints in \(C_d \cup C_u\) with their STP components.

Efficient solution methods for STPs are well-known [4]. A graphical representation of an STP is a Simple Temporal Network (STN), a graph of nodes representing the variables of the STP and edges labeled with the interval temporal constraints. Each STN is associated with a distance graph derived from the upper and lower bounds of the interval constraints. An STN is consistent if the distance graph does not contain a negative cycle; this condition can be determined by applying a single-source shortest path algorithm such as Bellman-Ford. In addition to consistency, it is often useful to determine for an STN the equivalent STN (in terms of a set of solutions) in which all the intervals are as “tight” as possible. This minimal network can be determined by applying an All-Pairs Shortest Path algorithm to the input network [4].

Previous efforts in solving STPPs have been based on identifying and applying criteria for “globally preferred solutions” such as “weakest link” (maximize the least preferred local preference), “pareto”, and “utilitarian” [10]. Developing efficient solvers has required local preference functions that are linear or semi-convex.\(^3\) One method for solving STPPs efficiently is called the chop method, first introduced in [9]. The chop method is a two-step search process of iteratively choosing a preference value \(\alpha\), “chopping” every preference function at that point, and then solving an underlying STP defined by the interval of temporal values whose preference values lie above the chop line, i.e. \(\{x : f(x) \geq \alpha\}\); henceforth, we refer to this as the chop interval. The highest chop point that results in a solvable (i.e. consistent) STP produces a flexible plan whose solutions are exactly the optimal solutions of the original STPP (based on the criteria of weakest link). Binary search can be used to select candidate chop points, making the technique for solving the STPP tractable.

\(^3\)A function is semi-convex if drawing a horizontal line anywhere in the Cartesian plane of the graph of the function is such that the set of \(X\) such that \(f(X)\) is not below the line forms an interval. Semi-convexity ensures that there is a single interval above any chop point, and hence that the resulting problem is an STP.
This technique can be generalized for arbitrary STP3s. Given an STP3 $P$, to determine the probability of achieving a solution of global preference value $\gamma$ or higher, we perform the following procedure:

1. Given an input STP3, chop each local preference function at the designated preference value $\gamma$. Form a new problem by replacing each associated interval with the resulting chop interval.

2. Determine the minimal network of the underlying STP of the new problem, using an All-Pairs Shortest Path algorithm.

3. Compute the overall probability of the underlying probabilistic CSP. Assuming independence of the $p_{i,j}$, the value to be computed is

$$\prod_{p_{i,j}} P(a_{i,j} \leq t \leq b_{i,j}),$$

where for each uncertainty constraint, $[a_{i,j}, b_{i,j}]$ is the interval of the minimal network derived from step 2.

Provided step 3, which may be done using numerical integration, is of polynomial complexity, the whole method is polynomial. Steps 2 and 3 of this method resemble the method proposed in [17] for solving Probabilistic STPs. Unfortunately, it can be easily shown that the computed value provides only an upper bound on the probability that the solutions defined at that chop level or above will succeed. That this is not a tight upper bound can be demonstrated by a simple example, found in Figure 3. In this example, chopping the preference function at 10 and solving the underlying STP would not shrink the temporal bounds of the uncertainty links. Therefore, the probability of succeeding returned by this method would be 1, although in fact some of the probability mass is lost as a result of the chop.

Despite these limitations, an upper bound computation may be useful; if the bound is too low, the planner will be forced to “lower expectations” of the plan branch under consideration, i.e. its overall expected preference level.

A tighter bound would require examining the mass of the polytope defined by all the constraints (a similar observation was made in [17]). Applied to the previous example, we get $P((0 \leq AB \leq 10) \land (0 \leq BC \leq 10))$ from (1), but the true probability is $P((0 \leq AB \leq 10) \land (0 \leq BC \leq 10) \land (AB + BC \geq 10))$ or simply $P(AB + BC \geq 10)$, assuming the bounds. (Note that the AB and BC random variables are no longer independent under the condition $AB + BC \geq 10$.) We can reformulate this as $P(\exists x (AB = x \land BC \geq 10 - x))$ and calculate it as

$$\int_{0}^{10} \left( \int_{10-x}^{10} p(y)dy \right) p(x)dx.$$

5 Inducing Preferences from Probabilities

In this section we consider a sort of dual problem to that posed in the previous section: given current expectations about the world, how can preferences be systematically adjusted to fit with those expectations? For example, a preference might be expressed for a particular gap between an uncontrollable event such as a volcano eruption, and a remote sensing event. There may also be a belief, expressed as a probability distribution, regarding when the volcano will occur. From these inputs, if may be possible to infer a set of preferred (high utility) start times for the observation.

The solution involves apply the concept of expected utility from decision analysis [8] to represent induced local preferences. Once the reasoning is complete, the “output” preferences on the decision constraints thus reflect both the preferences of the agent and its expectation about the uncertainty in the world. The solution consists of three steps:

1. Given an input STP3, derive the minimal network of the underlying STP.

2. Apply a local consistency algorithm (discussed below) to the resulting STP3 (i.e. with the tightened interval constraints) to compute the induced preferences.

3. Solve the underlying STPP of the resulting network using the chop solver to find the globally preferred solutions.

We refer to the set of solutions making up the flexible plan that results from this method as the expected globally preferred solutions.

To examine the second step in more detail, we mimic the method of triangular reduction found in [12], used to solve Simple Temporal Problems with Uncertainty (STPU). We consider all STP3s as collections of triangular subnetworks of the form illustrated by Figure 2(b), where there is a single uncertainty constraint on $AC$ with bounds $[u, v]$, and two decision constraints on $AB$ and $BC$ with bounds $[y, z]$ and $[w, x]$ respectively. As in the Earth Science example, $A$ might be the beginning of time, $B$ might be the start of a planned observation, and $C$ the onset of a fire. The goal is to compute the regression of $p_{AC}$ over $f_{BC}$ to find the induced soft decision constraint $f_{AB}$. (The case in which $AB$ is also associated with a soft constraint can be handled as part of the general solution method discussed later.)

To handle the single triangle case, we need to consider three possible orderings between $B$ and $C$. We assume that step 1 of the approach has been applied, so that the triangular network has been minimized. If $B$ precedes $C (w \geq 0)$, then
the induced soft constraint is $\{[y, z], f_{AB}\}$, where
$$f_{AB}(t) = \int_u f(t' - t)p(t')dt'.$$

Although in general this function cannot be derived analytically, with certain restrictions placed on the shape of the preference function it may be possible to compute it directly. Alternatively, we can estimate it numerically (e.g. Monte Carlo integration), or even perform crude but fast estimation based on the expected value. If $C$ precedes $B$ ($x \leq 0$), then intuitively the planner does not require any knowledge about the expected time of $C$ in order to deduce the preferred time to execute $B$ dynamically (the soft constraint on $AB$ in this case can be derived from that of $BC$). However, recall that we focus only on the situation of static execution, in which knowledge about $C$ is not available at planning time. This means that the predictive models of the Precede case are relevant to planning the Follow case: the same technique can be followed. Finally, for static execution the same also applies if $B$ and $C$ are unordered ($u < 0, x > 0$).

To derive the induced constraints for general STP$^3$ networks, we consider all triangles separately, propagating the effects of one operation to neighboring triangles, until the network is quiescent. Thus, the structure of the algorithm is similar to determining path-consistency in an STP network. Propagation requires an operation of combining local preference functions. The same combination operator as that used for determining local consistency for preference networks [14] can be applied here for propagating soft constraints. After the network has reached quiescence, the planner can safely discard the probability density functions $p_{XY}$ in $C_u$. Removing them results in the underlying STPP, which can be solved by the chop method [9].

The following result summarizes these core ideas. It will be proved informally and illustrated by an example. Following terminology in [12], an STP$^3$ will be said to be pseudo-controllable if no interval in an uncertainty constraint is “squeezed” as the result of performing step 1 above (computing the minimal network). We refer to the STP$^3$ that results from performing step 2 above as the induced STP$^3$.

**Theorem 1** Given an STP$^3$ with the following properties:

1. The input preference functions $p_{ij}$ are linear or semi-convex piecewise linear (intuitively, semi-convex piecewise linear means that there are no “V” shaped segments);
2. The STP$^3$ is pseudo-controllable;
3. The probability distributions on the uncertainty constraints are normal;
then, using the method described above, the set of expected globally preferred solutions to the initial STP$^3$ can be computed in polynomial time.

The first condition of the theorem is needed to ensure that the induced STP$^3$ has only functions that are semi-convex, which is required for the application of the chop solver method in step 3 (a polynomial-time procedure). Steps 2 and 3 are required to simplify the induced functions to linear functions involving expected values (see the example below). The conclusion of the proof consists of observing that the underlying procedures applied in the method (all-pairs shortest path, the local-consistency technique for deriving induced preferences, the chop solver, and numerical integration for determining the expected values) are all polynomial.

To illustrate step 2 of the method in the general case, consider the STP$^3$ in Figure 4. This problem consists of two decision constraints on $BC$ and $BD$ with associated preference functions $f, g$ defined, $f$ clearly preferring larger durations between $B$ and $C$, and $g$ preferring smaller durations. Two uncertainty constraints on $AC$ and $AD$ consist of normal probability density functions $p_1$ and $p_2$ with means and standard deviations indicated in parentheses. The goal is to infer the induced preference function $h$ on $AB$ (the network is already minimal).

First, considering the triangle $ABC$, one induced function for $h$ arises as follows:

$$h_1(t) = \int_0^1 f(t' - t)p_1(t')dt'$$
$$= \int_0^1 [t' - t]p_1(t')dt'$$
$$= \int_0^1 t'p_1(t')dt' - t\int_0^1 p_1(t')dt'.$$

Notice that because of the pseudo-controllability of the network (it being already minimal), the last equation reduces to $E(T_1) - t$, since then $\int_0^1 p_1(t')dt' = 1$ and $\int_0^1 t'p_1(t')dt' = E(T_1)$, where $E(T_1)$ is the expected value of the random variable $T_1$ associated with the duration. A similar derivation based on the triangle $ABD$ then results in another induced function $h_2(t) = 10 - [E(T_2) - t]$. The final induced function $h$ becomes the combination of $h_1$ and $h_2$: e.g. the intersection of the areas under the functions.

This approach can be generalized for regression over semi-convex piecewise linear preference functions. Let $f_{BC}$ be the intersection of $n$ linear segments $f_{BC}^1, \ldots, f_{BC}^n$, where for each $k$, $[a_{BC}^k, b_{BC}^k]$ is the segment for which $f_{BC} = f_{BC}^k$. When regressing $p_{AC}$ over $f_{BC}$ to compute the induced pref-

**Figure 4: Example of Induced Preferences**

![Diagram of Example of Induced Preferences]
structure function $h_{AB}$, we have:

$$h_{AB}(t) = \int_{a_1}^{b_1} f_{BC}(t' - t)p_{AC}(t')dt'$$

$$= \sum_{k=1,...,n} \int_{a_k}^{b_k} f_{BC}^k(t' - t)p_{AC}^k(t')dt',$$

which simplifies the calculation to sums involving linear functions.

This example shows how with suitable restrictions on the shapes of the preference functions and on whether all pairs computation eliminates any of the probability mass, the computation of induced preferences can be made efficient.

## 6 Discussion and Future Work

We have examined temporal reasoning under the interactions of preferences and quantitative uncertainty in the context of constraint-based planning. In addition to the formulation of the STP framework, which augments the Simple Temporal Problem with both preferences and probabilities, the main contribution of this paper is to formulate two planning decision problems. Utilizing standard methods from decision theory, probability theory, and recent advances in constraint satisfaction, we have shown how flexible temporal plans can be generated that are most preferred based on what the planning agent believes about the expected times of events; and how the agent can update its preferences, given its beliefs.

Fundamentally, preferences and uncertainty are orthogonal aspects of the decision problem. Both planning decisions we have examined are approaches to combining the two aspects; which is most relevant depends on the aim of the planning agent and the questions being asked of it. The first decision, to evaluate the probability of a plan existing with at least a given preference, is useful to determine whether a plan branch can meet a minimum quality threshold. The second decision, to update preferences based on beliefs, is useful to factor the uncertainty into a single criterion for plan evaluation. Besides these two decision problems, the proposed framework can be applied to related problems; for instance, an agent might seek to determine the maximal preference level at which a solution exists with a given probability $p$. When $p = 1$ and the probabilities are uniform, this corresponds to certain forms of strong controllability addressed in [15].

Future theoretical efforts include characterizing more fully the computational complexity of STP systems and refining the bound on the probability that a plan exists with given quality w.r.t. preference. In addition to implementing the methods described in this paper, our major next step is to extend the results here to address issues in planning under a dynamic execution strategy. Of particular importance will be to examine the interactions between preferences and wait constraints that emerge when determining the controllability of flexible plans, as described in [11].

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**References**


